CSIR-NET – MATHEMATICAL SCIENCES
DEC. 2024

PART - B (Mathematical Sciences)

UNIT-1

1. Let $U_1, U_2, ..., U_5$ be 5 urns such that urn U_k contains $2k + k^2$ balls, out of which $2k$ are white balls and k^2 are black balls, $k = 1, 2, ..., 5$. An urn is selected with probability of selecting urn U_k being proportional to $(k+2)$. A ball is chosen randomly from the selected urn. Then, the probability that the urn U_5 was selected, given that the ball drawn is white, is equal to

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{3}{4}$

Correct option is (c)

2. What is the number of injective functions from $\{1, 2, ..., 7\}$ to $\{1, 2, ..., 10\}$?

(a) 10^7 (b) $\frac{10!}{7!}$ (c) $\frac{10!}{3!}$ (d) 7^{10}

Correct option is (c)

3. Let S denote the set of all solutions of the Euler-Lagrange equation of the variational problem:

minimize $J[y] = \int_0^1 (y^2 + (y')^2)\,dx,$

subject to $y(0) = 0,\ y(1) = 0,\ \int_0^1 y^2 dx = 1.$

Then the set $\left\{\phi\left(\frac{1}{2}\right) : \phi \in S\right\}$ is equal to

(a) $\left\{-\sqrt{2}, \sqrt{2}\right\}$

(b) $\left\{\frac{\sqrt{2}}{k} : k \in \mathbb{Z},\ k \neq 0\right\}$

(c) $\left\{\sqrt{\frac{2}{k}} : k \in \mathbb{N}\right\}$

(d) $\left\{-\sqrt{2}, 0, \sqrt{2}\right\}$

Correct option is (d)

4. For a variable x, consider the $\mathbb{R}$ - vector space $V = \left\{a_0 + a_1 x + a_2 x^2 \middle| a_1, a_2, a_3 \in \mathbb{R}\right\}$. Let $T : V \to V$ be the linear transformation defined by $T(f) = f + \frac{df}{dx}$, where $\frac{df}{dx}$ denotes the derivative of f with respect to x. Which of the following statements is true ?

(a) $(T^3 - 3T^2 + 3T)^{2025}(x) = x$ (b) $(T^3 - 3T^2 + 3T)^{2025}(x) = x + 1$

(c) $(T^3 - 3T^2 + 3T)^{2025}(x) = 2025!\,x$ (d) $(T^3 - 3T^2 + 3T)^{2025}(x) = 2025!\,x + 1$

Correct option is (a)

5. Let V be the $\mathbb{R}$ - vector space of 5×5 real matrices. Let $S = \{AB - BA \mid A, B \in V\}$ and W denote the subspace of V spanned by S. Let $T : V \to \mathbb{R}$ be the linear transformation mapping a matrix A to its trace. Which of the following statements is true ?

(a) $W = \ker(T)$ (b) $W \subset \ker(T)$ (c) $W \cap \ker(T) \subset W$ (d) $W \cap \ker(T) \subset \ker(T)$

Correct option is (a)

6. Let P be the population proportion of units possessing a certain attribute in a population of N units. Let p be the sample proportion in a simple random sample (without replacement) of n units, $(2 \le n < N)$. Then an unbiased estimator of $P(1-P)$ is :

(a) $\dfrac{(N-n)}{Nn} p(1-p)$ (b) $\dfrac{(N-n)}{(N-1)n} p(1-p)$ (c) $\dfrac{n}{(n-1)} p(1-p)$ (d) $\dfrac{(N-n)n}{N(n-1)} p(1-p)$

Correct option is (d)

7. Let $\mathbb{D} = \{z = x + iy \in \mathbb{C} : |z| < 1\}$ be the open unit disc and $f : \mathbb{D} \to \mathbb{C}$ a holomorphic function such that $f(0) = 0$. Let $\psi(z) = |f(z)|^2$ and $\dfrac{\partial^2 \psi}{\partial x^2} + \dfrac{\partial^2 \psi}{\partial y^2} \equiv 0$. Which of the following statements is FALSE ?

(a) f can be extended to $\mathbb{C}$ as an entire function.

(b) f must have infinitely many zeros in $\mathbb{D}$.

(c) f is not a polynomial.

(d) $\exp(f)$ cannot take every complex value.

Correct option is (c)

8. We say that a group G has property (A) if every non-trivial homomorphism from G to any group is injective. Which of the groups has property (A) ?

(a) The cyclic group of order 6. (b) The symmetric group S_5.

(c) The alternating group A_5. (d) The dihedral group with ten elements.

Correct option is (c)

9. For integers $n > 1$, let $G(n)$ denote the number of groups of order n, up to isomorphism, i.e. $G(n)$ is the number of isomorphism classes of groups of order n. Which of the following statements is true ?

(a) If $G(n) = 1$, then n is prime.

(b) $G(8) = 2$

(c) If $\gcd(n, \phi(n)) > 1$, then $G(n) > 1$. (Here, ϕ denotes the Euler ϕ-function)

(d) $\limsup_{n\to\infty} G(n) = 2$

Correct option is (c)

10. $\lim\limits_{n\to\infty}\int_0^1\int_0^1\cdots\int_0^1 \frac{x_1^2+\cdots+x_n^2}{x_1+\cdots+x_n}\,dx_1\ldots dx_n$ equals

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{2}{3}$

Correct option is (d)

11. Which of the following statements is true ?

(a) $\left\{m+ne^{\frac{2\pi i}{3}} \,\middle|\, m, n\in\mathbb{Z}\right\}$ is a dense subset of $\mathbb{C}$.

(b) Open connected subsets of $\mathbb{R}^3$ need not be path connected.

(c) Let X be a topological space and $p: X\to\mathbb{R}$ a continuous surjective open map. If $p^{-1}(\{\alpha\})$ is connected for every $\alpha\in\mathbb{R}$, then X must be connected.

(d) Compact subsets of any infinite topological space are closed.

Correct option is (c)

12. Let X, Y and Z be random variables such that

$$S=\begin{pmatrix} X & Y \\ Y & Z\end{pmatrix}\sim W_2(10,\Sigma),$$

where W_2 denotes the Wishart distribution and

$$\Sigma=\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1\end{pmatrix}$$

Define $T=Z-\frac{Y^2}{X}$. Then, $\mathrm{Var}(T)$ equals

(a) $\frac{77}{6}$ (b) $\frac{81}{8}$ (c) $\frac{83}{9}$ (d) $\frac{79}{7}$

Correct option is (b)

13. Let $X_1, X_2,\ldots, X_n$ $(n\geq 2)$ be a random sample from a gamma distribution with shape parameter $\alpha>0$ and scale parameter $\beta=1$. For a suitable constant C, the rejection region of the most powerful test for testing $H_0:\alpha=1$ against $H_1:\alpha=2$ is of the form

(a) $\prod_{i=1}^n X_i > C$ (b) $\sum_{i=1}^n X_i > C$ (c) $\prod_{i=1}^n X_i < C$ (d) $\sum_{i=1}^n X_i < C$

Correct option is (...)

14. Two blocks of equal mass m are connected by a flexible inelastic cord of mass M. One block is placed on a smooth horizontal table, the other block hangs over the edge. The total potential energy of the entire cord is given by $\frac{-Mg}{2l}x^2$, where x is the distance of the hanging block from the edge of the table, l is the length of the cord, and g is the gravitational acceleration. Then,

(a) $\ddot{x} = \frac{l}{g}\frac{ml + Mx}{2m + M}$ (b) $\ddot{x} = \frac{l}{g}\frac{Ml + mx}{m + M}$ (c) $\ddot{x} = \frac{g}{l}\frac{ml + Mx}{m + M}$ (d) $\ddot{x} = \frac{g}{l}\frac{ml + Mx}{2m + M}$

Correct option is (d)

15. Consider the sequences $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ defined by:

$$a_n = \frac{e^n + e^{-n}}{2} \text{ and } b_n = \frac{a_{n+1}}{a_n}$$

Which of the following statements is true ?

(a) For each $x \in \mathbb{R}$, there exists an n such that $a_n > x$.

(b) For each $x \in \mathbb{R}$, there exists an n such that $a_n < x$.

(c) For each $x \in \mathbb{R}$, there exists an n such that $b_n > x$.

(d) For each $x \in \mathbb{R}$, there exists an n such that $b_n < x$.

Correct option is (a)

16. In an examination question paper, all questions are 'True' or 'False' type. These are arranged in such a way that three fourth of times a question with answer 'True' is followed by a question with answer 'True'. Also two-third of times a question with answer 'False' is followed by a question with answer 'False'. If the question paper has 100 questions, the approximate probability that the correct answer of the 100th question is 'True', is:

(a) $\frac{3}{7}$ (b) $\frac{4}{7}$ (c) $\frac{3}{4}$ (d) $\frac{5}{6}$

Correct option is (b)

17. Let $\lambda \in \mathbb{R}$, and $K : [0, 1] \times [0, 1] \to \mathbb{R}$ be a function such that every solution of the boundary value problem:

$$\frac{d^2u}{dx^2}(x) + \lambda u(x) = 0; \quad \frac{du}{dx}(0) = u(0), \quad \frac{du}{dx}(1) = 0$$

satisfies the integral equation, $u(x) + \lambda \int_0^1 K(x, t) u(t)\, dt = 0$

Then,

(a) $K(x, t) = \begin{cases} (1+x)(1-t), & 0 \leq x \leq t \leq 1, \\ (1+t)(1-x), & 0 \leq t < x \leq 1 \end{cases}$ (b) $K(x, t) = \begin{cases} -1-x, & 0 \leq x \leq t \leq 1, \\ -1-t, & 0 \leq t \leq x \leq 1 \end{cases}$

(c) $K(x, t) = \begin{cases} 1-x^2, & 0 \leq x \leq t \leq 1, \\ 1-t^2, & 0 \leq t < x \leq 1 \end{cases}$ (d) $K(x, t) = \begin{cases} (1+x)(t-1), & 0 \leq x \leq t \leq 1, \\ (1+t)(x-1), & 0 \leq t < x \leq 1 \end{cases}$

Correct option is (b)

18. For integers $m, n \geq 1$, let $I_{m,n} = \frac{1}{2\pi i}\int_C z^m \bar{z}^n\, dz$, where C is the circle $\{z \in \mathbb{C} : |z| = 1\}$ oriented counter-clockwise. Which of the following statements is true ?

(a) $I_{m,n} = 1$, if $m = n$ (b) $I_{m,n} = 1$, if $m + 1 = n$

(c) $I_{m,n} = 1$, if $m = n + 1$ (d) $I_{m,n} = 1$, if $m = n + 2$

Correct option is (b)

19. Let $f:[0,1]\to\mathbb{R}$ be defined by $f(x)=\sin(x^2)$. Let $A=\lim_{n\to\infty}\left(\sum_{k=1}^{n} f\left(\frac{k}{n}\right)-n\int_0^1 f(x)\,dx\right)$. Which of the following statements is true ?

(a) $A=0$ (b) $A=1$ (c) $A=\frac{\sin(1)}{2}$ (d) $A=\sin\left(\frac{1}{4}\right)$

Correct option is (c)

20. Let $f:\mathbb{R}\to\mathbb{R}$ be such that $\sup_{x\neq y}\frac{|f(x)-f(y)|}{|x-y|}=L$, where $1<L<\infty$. Let $h:\mathbb{R}\to\mathbb{R}$ be a differentiable function satisfying $|h'(x)|\le\frac{3}{4}$ for all $x\in\mathbb{R}$. For $\alpha>0$, define $g(x)=\alpha f(x)+h(x)$ for $x\in\mathbb{R}$. Consider the sequence $\{x_k\}_{k=0}^{\infty}$ defined by $x_{k+1}=g(x_k),\ k=0,1,\ldots,$ where $x_0\in\mathbb{R}$. The sequence $\{x_k\}_{k=0}^{\infty}$ converges to the solution of the equation $x=g(x)$ if

(a) $\alpha<\frac{2}{3L}$ (b) $\alpha<\frac{3}{2L}$ (c) $\alpha<4L$ (d) $\alpha<\frac{1}{4L}$

Correct option is (d)

21. Let $v=(a,b,c)\in\mathbb{R}^3$ be a non-zero vector that lies in the orthogonal complement (with respect to the standard inner product) of the row-space of the matrix:

$$A=\begin{pmatrix}2 & 2 & 7\\ 3 & 1 & 4\end{pmatrix}$$

If a,b,c are all integers, then what is the smallest possible value of $|a+b+c|$?

(a) 5 (b) 10 (c) 15 (d) 20

Correct option is (b)

22. Let $\mathbb{C}[x,y]$ be the polynomial ring in two variables over $\mathbb{C}$. For which of the following ideals I, the quotient ring $\mathbb{C}[x,y]/I$ is NOT an integral domain ?

(a) $I=(x,y)$ (b) $I=(x+y)$ (c) $I=(x^2+y^2)$ (d) $I=(xy-1)$

Correct option is (c)

23. Let $u=u(x,t)$ be a solution of the wave equation:

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=0,\ x\in\mathbb{R},\ t>0$$

satisfying the condition $u(0,t)=0,\ \forall\ t\ge 0$. Then which of the following statements is true ?

(a) $u(x,t)=0$, whenever $x=t$ (b) $u(x,t)=0$, whenever $x=-t$

(c) $u(-x,t)=u(x,t)$, whenever $x>0,t>0$ (d) $u(-x,t)=-u(x,t)$ whenever $0<x\le t$

Correct option is (d)

24. Let $A=\begin{bmatrix}0 & a & 0\\ 0 & 0 & b\\ c & 0 & 0\end{bmatrix}$, where a, b, c are real numbers with $abc=1$. If $B=A+A^2+A^3$, then which of the following statements is true ?

(a) $\det B=1$ (b) $\det A=0$ (c) $\operatorname{rank}(B)=2$ (d) $\operatorname{rank}(B^2)=1$

Correct option is (d)

25. **Stats. Question ....**

Correct option is ()

26. **Stats. Question ....**

Correct option is ()

27. Suppose that the differential equation:

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+e^{2x}y=0,\ x\in\mathbb{R}$$

transforms into a second order differential equation with constant coefficients under the change of independent variable given by $s=s(x)$ satisfying $\frac{ds}{dx}(0)=1$. Then which of the following statements is true ?

(a) $e^{-x}(P(x)+1)$ is a constant function on $\mathbb{R}$. (b) $e^{-2x}P(x)$ is a constant function on $\mathbb{R}$.

(c) $s(x)=\frac{e^{2x}}{2},\ x\in\mathbb{R}$ (d) $P(x)\to 1$ as $x\to\infty$

Correct option is (a)

28. Consider the bilinear form $B:\mathbb{R}^4\times\mathbb{R}^4\to\mathbb{R}$ defined by $B(x,y)=x_1y_3+x_2y_4-x_3y_1-x_4y_2$, where $x=(x_1,x_2,x_3,x_4)$ and $y=(y_1,y_2,y_3,y_4)$ in $\mathbb{R}^4$. Let A denote the matrix of B with respect to the standard ordered basis of $\mathbb{R}^4$. Which of the following statements is true ?

(a) $\det A=0$

(b) $\det A=-1$

(c) $B(x,x)\neq 0$ for all non-zero $x\in\mathbb{R}^4$

(d) If $x\in\mathbb{R}^4$ is non-zero, then there exists $y\in\mathbb{R}^4$ such that $B(x,y)\neq 0$

Correct option is (d)

29. Consider the power series:

$$\sum_{n=1}^{\infty}\frac{n^{n^2}}{(n+1)^{n^2}}x^n$$

with coefficients in real numbers $\mathbb{R}$. Which of the following statements is true ?

(a) The radius of convergence of the series is $\frac{1}{e}$.

(b) The series converges at $x = 5$.

(c) The series converges at $x = 3$.

(d) The series converges for all x with $|x| < \frac{1}{2}$.

Correct option is (d)

30. **Stats. Question ....**

Correct option is ()

31. Given that $y_1(x) = e^{2x}$ is a solution of the ordinary differential equation (ODE)

$$x\frac{d^2y}{dx^2} - (3+4x)\frac{dy}{dx} + (4x+6)\,y = 0,\ x > 0.$$

Let $y_2 = y_2(x)$ be the solution of the ODE satisfying the conditions $y_2(1) = \frac{e^2}{4}$, $\frac{dy_2}{dx}(1) = \frac{3e^2}{2}$. Then which of the following statements is true ?

(a) y_2 is a strictly increasing function on $(0, \infty)$.

(b) $e^{-2x}y_2(x) \to 1$ as $x \to \infty$

(c) y_2 is a strictly decreasing function on $(0, \infty)$

(d) $e^{-2x}y_2(x) \to 0$ as $x \to \infty$

Correct option is (a)

32. Let $f : \mathbb{C} \to \mathbb{C}$ be the function defined by:

$$f(z) = e^{(\cos(1+i))\sin z}$$

For $z = x + iy \in \mathbb{C}$, write $f(z)$ as $u(x, y) + iv(x, y)$, where u, v are real-valued functions. Which of the following is the value of $\frac{\partial u}{\partial x}(0, 0)$?

(a) 0 (b) $\left(e + \frac{1}{e}\right)\frac{\cos 1}{2}$ (c) $\left(e - \frac{1}{e}\right)\frac{\cos 1}{2}$ (d) 1

Correct option is (b)

33. Let $u = u(x, y)$ be the solution of the Cauchy problem:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u,\ (x, y) \neq (0, 0);\quad u(x, 1) = \sqrt{1 + x^2},\ x \in \mathbb{R}$$

Then which of the following statements is true ?

(a) $u(1, 0) = 0$

(b) $u(x_1, y_1) = u(x_2, y_2)$ whenever $x_1^2 + y_1^2 = x_2^2 + y_2^2$

(c) $u(1, y) = \sqrt{2}$ for all $y \in \mathbb{R}$

(d) $u(x_1, y_1) = u(x_2, y_2)$ whenever $x_1 + y_1 = x_2 + y_2$

Correct option is (b)

34. Let $\mathbb{H} = \{z = x + iy \in \mathbb{C} \mid y > 0\}$ and $f : \mathbb{H} \to \mathbb{C}$ be a non-constant holomorphic function satisfying $|f(z)| < 1$ for all $z \in \mathbb{H}$. Which of the following statements is true ?

(a) $\lim\limits_{y \to +\infty} f'(iy) = 0$

(b) $\lim\limits_{y \to +\infty} f'(iy)$ is a complex number with absolute value 1

(c) $\lim\limits_{y \to +\infty} |f'(iy)| = +\infty$

(d) $\lim\limits_{y \to +\infty} f'(iy)$ is not a real number

Correct option is (a)

35. For integers $n \geq 0$, let $f_n : [-1, 0] \to \mathbb{R}$ be defined by

$$f_n(x) = \frac{x}{(1-x)^n}$$

Which of the following statements is true about the series $\sum\limits_{n=0}^{\infty} f_n$?

(a) The series is neither absolutely convergent nor uniformly convergent.

(b) The series is both absolutely convergent and uniformly convergent.

(c) The series is absolutely convergent but not uniformly convergent.

(d) The series is uniformly convergent but not absolutely convergent.

Correct option is (c)

36. Let U denote the span of $\{e^t, e^{2t}, e^{3t}\}$ in the real vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Consider the $\mathbb{R}$ - vector spaces

$V = \{f : U \to \mathbb{R} \mid f$ is an $\mathbb{R}$-linear transformation$\}$

$W = \{f \in V \mid f(e^{3t}) = 0\}$.

Which of the following statement is true ?

(a) Both V and W are infinite-dimensional.

(b) $\dim V = 3$ and $\dim W = 1$.

(c) $\dim V = 3$ and $\dim W = 2$.

(d) V is infinite-dimensional and $\dim W = 0$.

Correct option is (c)

37. **Stats. Question ....**

Correct option is ()

38. Suppose that $X \sim$ binomial $(9, \theta), 0.7 < \theta < 1$, and $Y \sim$ Poisson $(\lambda), \lambda > 0$. If $3E(Y) = E(X)$, then which of the following is true ?

(a) $\text{Var}(X) > 3\text{Var}(Y)$ (b) $2\text{Var}(Y) < \text{Var}(X) < 3\text{Var}(Y)$

(c) $\text{Var}(Y) < \text{Var}(X) < 2\text{Var}(Y)$ (d) $\text{Var}(X) < \text{Var}(Y)$

Correct option is (d)

39. Let A, B and C be sets. Which of the following sets is equal to $A \setminus (B \setminus C)$?

(a) $A \setminus B$ (b) $(A \setminus B) \cup C$ (c) $A \setminus (B \cup C)$ (d) $(A \setminus B) \cup (A \cap C)$

Correct option is (d)

40. **Stats. Question ....**

Correct option is ()

PART - B (Mathematical Sciences)

1. Let disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and f be a holomorphic function on $\mathbb{D}$ such that the function $g(z) = e^{1/z} f(z)$ on $\mathbb{D} \setminus \{0\}$ is bounded. Which of the following statements are true ?

(a) $f(0) = 0$

(b) $f(z) = 0$ for all $z \in \mathbb{D}$

(c) There exists a nonzero constant c such that $f(z) = ce^{-1/z}$ for all $z \in \mathbb{D} \setminus \{0\}$

(d) There exists a nonzero constant c and a positive integer n such that $f(z) = cz^n e^{-1/z}$ for all $z \in \mathbb{D} \setminus \{0\}$

Correct options are (a), (b)

2. The integral equation $u(x) = f(x) + \dfrac{2}{\pi}\displaystyle\int_0^{\pi} \sin(x-t)\,u(t)\,dt$ has a unique solution if

(a) $f(x) = \cos x$ (b) $f(x) = \cos 5x$ (c) $f(x) = \sin x$ (d) $f(x) = \sin 5x$

Correct options are (a), (b), (c), (d)

3. Let R be a nonzero ring with unity such that $r^2 = r$ for all $r \in R$. Which of the following statements are true?

(a) R is never integral domain. (b) $r = -r$ for all $r \in R$.

(c) Every nonzero prime ideal of R is maximal (d) R must be a commutative ring

Correct options are (b), (c), (d)

4. Consider $X = \{u \mid u : [0,1] \to \mathbb{R}$ is continuous and $u(0) = 0\}$ with the sup norm $\|u\| = \sup_{x \in [0,1]} |u(x)|$. Let $T(u) = \int_0^1 u(t)\,dt$ and $S = \{|T(u)| : u \in X, \|u\| \le 1\}$. Which of the following statements are true ?

(a) S is an unbounded subset of $\mathbb{R}$.

(b) S is a bounded subset of $\mathbb{R}$ and $\sup(S) = 1$.

(c) There exists $u \in X$ such that $\|u\| = 1$ and $T(u) = 1$.

(d) S is a closed subset of $\mathbb{R}$.

Correct option is (b)

5. For a variable x, consider the $\mathbb{Q}$-vector space $V = \left\{ax^3 + bx^2 + cx + d \mid a, b, c, d \in \mathbb{Q}\right\}$.

Further, let $A = \{ f : V \to \mathbb{Q} \mid f \text{ is a } \mathbb{Q}\text{-linear transformation} \}$, and $B = \{ f \in A \mid f(1) = 0 \}$

Which of the following statements are true ?

(a) If $f \in B$, then $\dim \ker f = 3$

(b) $\dim B = 3$

(c) $\dim A = 4$

(d) If $f \in A$, then the image of f is a one-dimensional $\mathbb{Q}$-vector space.

Correct options are (b), (c)

6. Define a topology τ on $\mathbb{R}$ as follows a subset U of $\mathbb{R}$ is in the topology τ if and only if $U = \phi$ or $0 \in U$. Which of the following statements are true ?

(a) The set of all irrational numbers is dense in $(\mathbb{R}, \tau)$.

(b) For each prime number p, the set $\left\{0, \sqrt{p}\right\}$ is dense in $(\mathbb{R}, \tau)$.

(c) $[0, 1]$ is compact in $(\mathbb{R}, \tau)$.

(d) $(\mathbb{R}, \tau)$ is Hausdorff.

Correct option is (b)

7. Consider the Markov chain with state space $\{0, 1, 2\}$ and the transition probability matrix P given by:

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Let $P^{(n)} = \left(\left(P_{ij}^{(n)}\right)\right)$ denote the n-step transition probability matrix. Then, which of the following statements are true ?

(a) $P_{00}^{(2)} = \dfrac{3}{4}$

(b) $P_{10}^{(3)} = \dfrac{39}{64}$

(c) The stationary probability that the chain is in state 2 is $\dfrac{2}{7}$

(d) State 1 is transient

Correct options are (a), (b), (c)

8. Consider the function $f:\mathbb{R}^2 \to \mathbb{R}$ defined by:

$$f(x,y)=\begin{cases} \dfrac{x^2y}{x^4+y^2}+e^{xy} & ; \quad (x,y)\neq(0,0) \\ 1 & ; \quad (x,y)=(0,0) \end{cases}$$

Which of the following statements are true ?

(a) f is differentiable on $\mathbb{R}^2\backslash\{(0,0)\}$.

(b) All the directional derivatives of f exist at (0, 0).

(c) f is differentiable on $\mathbb{R}^2$.

(d) f is not continuous at (0, 0).

Correct options are (a), (b), (d)

9. Consider the non-homogeneous ordinary differential equation (ODE)

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=\sin(e^{-5x}),\ x>0$$

Then which of the following statements are true ?

(a) Every solution of the ODE is bounded on $(0,\infty)$.

(b) There exists a solution of the ODE which is unbounded on $(0,\infty)$.

(c) Every solution of the ODE is unbounded on $(0,\infty)$.

(d) Every solution of the ODE tends to zero as $x\to\infty$.

Correct options are (a), (d)

10. Let $f\in\mathbb{R}[x]$ be a product of distinct monic irreducible polynomials $P_1, P_2, ..., P_n$, where $n\geq 2$. Let (f) denote the ideal generated by f in the ring $\mathbb{R}[x]$. Which of the following statements are true ?

(a) $\mathbb{R}[x]/(f)$ is a field.

(b) $\mathbb{R}[x]/(f)$ is a finite dimensional $\mathbb{R}$ - vector space.

(c) $\mathbb{R}[x]/(f)$ is a direct sum of fields, each of which is isomorphic to $\mathbb{R}$ or $\mathbb{C}$.

(d) There are no non-zero elements $a\in\mathbb{R}[x]/(f)$ such that $u^m=0$ for some $m\geq 1$.

Correct options are (b), (c), (d)

11. Define $S:=\left\{y\in C^1[-1,1]: y(-1)=-1,\ y(1)=3\right\}$. Let ϕ be the extremal of the functional $J:S\to\mathbb{R}$ given by $J[y]=\int_{-1}^{1}\left[(y')^3+(y')^2\right]dx$. Define $\|y\|_\infty := \max\limits_{x\in[-1,1]}|y(x)|$ for every $y\in S$ and let

$$B_0(\phi,\varepsilon):=\left\{y\in S:\|y-\phi\|_\infty<\varepsilon\right\},\ B_1(\phi,\varepsilon):=\left\{y\in S:\|y'-\phi'\|_\infty<\varepsilon\right\}$$

Then which of the following statements are true ?

(a) $\phi(x)=2x+1$ for every $x\in[-1,1]$

(b) There exists $\varepsilon>0$ such that $J[y]\geq J[\phi]$ for every $y\in B_0(\phi,\varepsilon)$.

(c) There exists $\varepsilon>0$ such that $J[y]\geq J[\phi]$ for every $y\in B_1(\phi,\varepsilon)$.

(d) There exists $\varepsilon>0$ such that $J[y]\leq J[\phi]$ for every $y\in B_1(\phi,\varepsilon)$.

Correct options are (a), (c)

12. A group G is said to be divisible if for every $y \in G$ and for every positive integer n, there exists $x \in G$ such that $x^n = y$. Which of the following groups are divisible ?

(a) $\mathbb{Q}$ with ordinary addition.
(b) $\mathbb{C}\backslash\{0\}$ with ordinary multiplication.
(c) The cyclic group of order 5.
(d) The symmetric group S_5.

Correct options are (a), (b)

13. Let $f(x)$ be the polynomial of degree at most 2 that interpolates the data $(-1, 2), (0, 1)$ and $(1, 2)$. If $g(x)$ is a polynomial of degree at most 3 such that $f(x) + g(x)$ interpolates the data $(-1, 2), (0, 1), (1, 2)$ and $(2, 17)$, then

(a) $f(5) + g(3) = 50$ (b) $2f(5) - g(3) = 4$ (c) $f(1) + g(3) = 50$ (d) $f(5) + g(3) = 74$

Correct options are (b), (c), (d)

14. Consider the system of two particles with total kinetic energy $T = \frac{5}{2}\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta}\cos\theta$, and Lagrangian $L = \frac{5}{2}\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta}\cos\theta + 2gl\cos\theta$, where x, θ are generalized coordinates, and g, l are positive constants. Then the non-zero frequency of the normal mode of the system with small oscillation $\left(|\theta| \ll 1\right)$ is:

(a) $\frac{5}{3}\sqrt{\frac{g}{l}}$ (b) $\sqrt{\frac{5g}{3l}}$ (c) $\frac{5}{2}\sqrt{\frac{g}{l}}$ (d) $\sqrt{\frac{5g}{2l}}$

Correct option is (b)

15. Let V be the $\mathbb{R}$ - vector space of real valued continuous functions on the interval $[0, \pi]$ with the inner product given by $\langle f, g\rangle = \int_0^\pi f(x)\,g(x)\,dx$. Let $S = \left\{\sin(x), \cos(x), \sin^2(x), \cos^2(x)\right\}$ and W be the subspace of V generated by S. Which of the following statements are true ?

(a) S is an basis of W.
(b) S is an orthonormal basis of W.
(c) There exist $f, g \in S$ such that $\langle f, g\rangle = 0$.
(d) S contains an orthonormal basis of W.

Correct options are (a), (c)

16. Let $P(z)$ be a non-constant polynomial over $\mathbb{C}$. Given $R > 0$, let $S_R = \left\{z \in \mathbb{C} : |P(z)| < R\right\}$. Which of the following statements are true ?

(a) S_R is an open subset of $\mathbb{C}$.
(b) S_R is a bounded subset of $\mathbb{C}$.
(c) $|P(z)| = R$ for every z on the boundary of S_R.
(d) Every connected component of S_R contains a zero of $P(z)$.

Correct options are (a), (b), (c), (d)

17. For a positive real number a, $\sqrt{a}$, denotes the positive square root of a. Consider the function $f:\mathbb{R}^2 \to \mathbb{R}$ defined by :

$$f(x, y) = \begin{cases} \dfrac{x}{|x|}\sqrt{x^2+y^2} & ; \quad x \neq 0 \\ 0 & ; \quad x = 0 \end{cases}$$

Which of the following statements are true ?

(a) f is continuous at (0, 0).

(b) The partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ exist at (0, 0).

(c) f is differentiable at (0, 0).

(d) f is not differentiable at (0, 0).

Correct options are (a), (b), (d)

18. **Stats. Question ....**

Correct option is ()

19. If $\lambda \in \mathbb{R}$ and $p \in \mathbb{R}$ are such that the quadrature formula:

$$\int_{x_0}^{x_0+h} f(x)\,dx \approx \lambda h(f(x_0) + f(x_0+h)) + ph^3(f''(x_0) + f''(x_0+h))$$

is exact for all polynomials of degree as high as possible, then

(a) $2\lambda + 24p = 0$ (b) $7\lambda - 12p = 4$ (c) $2\lambda + 24p = -3$ (d) $7\lambda - 12p = 11$

Correct options are (a), (b)

20. **Stats. Question ....**

Correct option is ()

21. Consider maximizing the objective function

$$P = x_1 + x_2$$

subject to

$$x_1 + x_2 + 2x_3 \le 5$$

$$2x_1 - 3x_3 \ge 1$$

$$x_2 + x_3 \le 0$$

$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$$

Then, which of the following statements are true ?

(a) The optimal solution is 4.

(b) An optimal point is (4, 1, 0).

(c) The optimal solution is 6.

(d) $\left(\dfrac{1}{2}, 0, 0\right)$ is a corner point.

Correct option is (d)

22. Let X_1 and X_2 be random variables having absolutely continuous distribution functions. Let $h_i(t)$ denote the hazard function of $X_i, i=1,2$. If $h_1(t) \le h_2(t)$ for all $t \in \mathbb{R}$, then which of the following statements are true?

(a) $P(X_1 > 1) \ge P(X_2 > 1)$

(b) $P(X_1 > 1) \le P\left(X_2 > \frac{1}{2}\right)$

(c) $E(X_1) \ge E(X_2)$ provided both the expectations exist

(d) $h(t) = h_1(t) + h_2(t),\ t \in \mathbb{R}$, is the hazard function of the random variable $Y = \min\{X_1, X_2\}$

Correct options are (a), (c)

23. For $b \in \mathbb{R}$, let $y_b = y_b(x)$ be the unique solution of the initial value problem

$$\frac{dy}{dx} = y^5 + y^4 + y^3 + y^2 + y + 1,\ \ y(0) = b$$

defined on its maximal interval of existence I_b. Then which of the following statements are true?

(a) There exists an $\alpha \in (0, \infty)$ such that for every $b \in \mathbb{R}$ with $b > \alpha$, the solution y_b is bounded above on I_b.

(b) There exists an $\alpha \in (0, \infty)$ such that for every $b \in \mathbb{R}$ with $b > \alpha$, the solution y_b is bounded below on I_b.

(c) There exists an $\alpha \in (-\infty, 0)$ such that for every $b \in \mathbb{R}$ with $b < \alpha$, the solution y_b is bounded above on I_b.

(d) There exists an $\alpha \in (-\infty, 0)$ such that for every $b \in \mathbb{R}$ with $b < \alpha$, the solution y_b is bounded below on I_b.

Correct options are (b), (c)

24. Let A, B, C be topological spaces such that A is homeomorphic to B, B is a subspace of C and $\overline{B} = C$. Let C be homeomorphic to a subspace W of A. Which of the following statements are FALSE ?

(a) The spaces $B, \overline{W}, C$ are homeomorphic.

(b) The spaces B, W, C are homeomorphic.

(c) If C is compact, then A, B, C are homeomorphic.

(d) If A is connected, then B and C are connected.

Correct options are (a), (b), (c)

25. If u is the solution of the Volterra integral equation:

$$u(x) = 3 + \sin x + \int_0^x \frac{3+\sin x}{3+\sin t} u(t)\, dt,$$

then

(a) $u\left(\frac{\pi}{2}\right) = 4e^{\pi/2}$ (b) $u(\pi) = 3e^{\pi}$ (c) $u(-\pi) = 4e^{-\pi}$ (d) $u\left(-\frac{\pi}{2}\right) = 4e^{\pi}$

Correct options are (a), (b)

26. Consider the polynomial $f(x) = x^{2025} - 1$ over $\mathbb{F}_5$, where $\mathbb{F}_5$ is the field with five elements. Let S be the set of all roots of f in an algebraic closure of the field $\mathbb{F}_5$. Which of the following statements are true ?

(a) S is a cyclic group.

(b) S has $\phi(2025)$ elements, where ϕ denotes the Euler ϕ-function.

(c) S has $\phi(2025)$ generates, where ϕ denotes the Euler ϕ-function.

(d) S has 81 elements.

Correct options are (a), (d)

27. For each positive integer n, define $f_n : [0, 1] \to \mathbb{R}$ by $f_n(x) = nx(1-x)^n$. Which of the following statements are true ?

(a) $(f_n)_{n\geq 1}$ does not converge pointwise on [0, 1].

(b) $(f_n)_{n\geq 1}$ converges pointwise to a continuous function on [0, 1].

(c) $(f_n)_{n\geq 1}$ converges pointwise to a discontinuous function on [0, 1].

(d) $(f_n)_{n\geq 1}$ does not converge uniformly on [0, 1].

Correct options are (b), (d)

28. Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Which of the following statements are true ?

(a) Both A and B are diagonalizable over $\mathbb{R}$.

(b) A is diagonalizable over $\mathbb{C}$ but not over $\mathbb{R}$.

(c) Neither A nor B is diagonalizable over $\mathbb{R}$, but both A and B are diagonalizable over $\mathbb{C}$.

(d) Neither A nor B is diagonalizable over $\mathbb{C}$.

Correct options are (b), (c)

29. Let (X_1, X_2) be a bivariate normal random vector with $E(X_1) = 1$, $E(X_2) = 0$, $Var(X_1) = 1$, $Var(X_2) = 1$, and correlation coefficient $\frac{1}{2}$. Let U be a $U(0, 1)$ random variable, which is independent of (X_1, X_2).

If $Z = \dfrac{UX_1 + X_2 - U}{\sqrt{U^2 + U + 1}}$, then which of the following statements are true ?

(a) The distribution of Z is symmetric about 0.

(b) $E(Z^2) = 2$

(c) $Var(Z^2) = 1$

(d) Z and U are independent random variables.

Correct option is (...)

30. For a 4×4 positive definite real symmetric matrix A and real numbers a, b, c, d, consider the 5×5 matrix

$$H = \left(\begin{array}{c|cccc} 0 & a & b & c & d \\ \hline a & & & & \\ b & & A & & \\ c & & & & \\ d & & & & \end{array}\right)$$

Which of the following statements are necessarily true ?

(a) $\det(B) > 0$ for every non-zero $(a, b, c, d) \in \mathbb{R}^4$.

(b) $\det(B) > 0$ for infinitely many $(a, b, c, d) \in \mathbb{R}^4$.

(c) $\det(B) \le 0$ for every $(a, b, c, d) \in \mathbb{R}^4$.

(d) $\det(B) \le 0$ for infinitely many $(a, b, c, d) \in \mathbb{R}^4$.

Correct options are (c), (d)

31. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) = 0$ for all $x \le 0$ and for all $x \ge 1$. Define,

$$F(x) = \sum_{n=-\infty}^{\infty} f(x+n), \quad x \in \mathbb{R}$$

Which of the following statements are true ?

(a) F is bounded.

(b) F is continuous on $\mathbb{R}$.

(c) F is uniformly continuous on $\mathbb{R}$.

(d) F is not uniformly continuous on $\mathbb{R}$.

Correct options are (a), (b), (c)

32. If $x = x(t),\ y = y(t)$ is the solution of the initial value problem

$$\frac{dx}{dt} = x - 4e^{-2t} y, \qquad \frac{dy}{dt} = e^{2t} x - y, \qquad x(0) = 1,\ y(0) = 1,$$

then which of the following statements are true ?

(a) $\lim_{t \to \infty} t^{-2} x(t)\, y(t) = 0$

(b) $x(1) = 0,\ y\left(\frac{1}{2}\right) = 0$

(c) $x\left(\frac{1}{2}\right) = 0,\ y(1) = 0$

(d) $\lim_{t \to \infty} t^{-2} x(t)\, y(t) = 2$

Correct options are (c), (d)

33. For every integer $n \ge 2$, consider a $\mathbb{C}$ - linear transformation $T : \mathbb{C}^n \to \mathbb{C}^n$. Let V be a subspace of $\mathbb{C}^n$ such that $T(V) \subseteq V$. Which of the following statements are necessarily true ?

(a) There exists a subspace W of $\mathbb{C}^n$ such that $\mathbb{C}^n = V + W$ and $V \cap W = \{0\}$

(b) There exists a subspace W of $\mathbb{C}^n$ such that $T(W) \subseteq W$, $\mathbb{C}^n = V + W$ and $V \cap W = \{0\}$

(c) Suppose that there exists a positive integer k such that T^k is the identity map. Then there exists a subspace W of $\mathbb{C}^n$ such that $T(W) \subseteq W$, $\mathbb{C}^n = V + W$ and $V \cap W = \{0\}$.

(d) Suppose that there exists a subspace W of $\mathbb{C}^n$ such that $T(W) \subseteq W$, $\mathbb{C}^n = V + W$ and $V \cap W = \{0\}$.

Then there exists a positive integer k such that T^k is the identity map.

Correct options are (a), (c)

34. **Stats. Question ....**

Correct option is ()

35. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by :

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right), & \text{if } \quad x \neq 0 \\ 0, & \text{if } \quad x = 0 \end{cases}$$

Which of the following statements are true ?

(a) $\lim\limits_{x \to 0} f(x)$ exists

(b) f is continuous at 0

(c) f is differentiable at 0

(d) $\lim\limits_{x \to 0} f'(x)$ does not exist.

Correct options are (a), (b), (c), (d)

36. **Stats. Question ....**

Correct option is ()

37. For a variable x, consider the real vector space $V = \left\{ax^3 + bx^2 + cx + d \mid a, b, c, d \in \mathbb{R}\right\}$. Let $D : V \to V$ be the linear transformation where $D(f)$ is the derivative of f with respect to x, and $M : V \to V$ be the linear transformation $M(f) = xD(f)$. Which of the following statements are true ?

(a) $DM \neq MD$

(b) $D + M$ is invertible

(c) DM is invertible

(d) $\text{rank}(DM) = \text{rank}(MD)$

Correct option is (a)

38. **Stats. Question ....**

Correct option is ()

39. **Stats. Question ....**

Correct option is ()

40. **Stats. Question ....**

Correct option is ()

41. Let $u = u(x, t)$ be the solution of the initial-boundary value problem:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \ (x, t) \in (0, 1) \times (0, \infty),$$

$$u(x, 0) = 4x(1 - x), \ x \in [0, 1],$$

$$u(0, t) = u(1, t) = 0, \ t \geq 0$$

Then which of the following statements are true ?

(a) $\lim_{t\to\infty} u(x,t) = 0$ for all $x \in (0,1)$

(b) $u(x,t) = u(1-x,t)$ for all $x \in (0,1),\ t > 0$

(c) $\int_0^1 (u(x,t))^2\,dx$ is a non-increasing function of *t*.

(d) $\int_0^1 (u(x,t))^2\,dx$ is a non-decreasing function of *t*.

Correct options are (a), (b), (c)

42. Let $(a_n)_{n\ge1}$, $(b_n)_{n\ge1}$ and $(c_n)_{n\ge1}$ be sequences given by:

$a_n = (-1)^n(1+e^{-n}),\quad b_n = \max\{a_1,\ldots,a_n\}$, and $c_n = \min\{a_1,\ldots,a_n\}$.

Which of the following statements are true ?

(a) $(a_n)_{n\ge1}$ does not converge

(b) $\limsup_{n\to\infty} a_n = \lim_{n\to\infty} b_n$

(c) $\liminf_{n\to\infty} a_n = \lim_{n\to\infty} c_n$

(d) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n$

Correct option is (a)

43. Let *A*, *B* be 2 × 2 matrices with real entries, and $M = AB - BA$. Let I_2 denote the 2 × 2 identity matrix. Which of the following statements are necessarily true ?

(a) If *A* and *B* are upper triangular, then *M* is diagonalizable over $\mathbb{R}$.

(b) If *A* and *B* are diagonalizable over $\mathbb{R}$, then *M* is diagonalizable over $\mathbb{R}$.

(c) If *A* and *B* are diagonalizable over $\mathbb{R}$, then there exists $\lambda \in \mathbb{R}$ such that $M = \lambda I_2$.

(d) There exists $\lambda \in \mathbb{R}$ such that $M^2 = \lambda I_2$.

Correct option is (d)

44. Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous function. Define $f(x) = \int_0^x (x-t)\,g(t)\,dt,\ x \in \mathbb{R}$. Which of the following statements are true?

(a) $f(0) = 0$

(b) $f'(0)$ exists and $f'(0) = 0$

(c) $f''(0)$ exists and $f''(0) = g(0)$

(d) $f''(0)$ exists but $f''(0) \neq g(0)$

Correct options are (a), (b), (c)

45. Let $f : \mathbb{C}\setminus\{-1,1\} \to \mathbb{C}$ be a holomorphic function that does not take any value in the set $\{z \in \mathbb{C} : |z-1| < 1\}$. Which of the following statements are true ?

(a) f is constant.

(b) f has removable singularities at –1 and 1.

(c) f is bounded.

(d) f has either poles or essential singularities at –1 and 1.

Correct options are (a), (b), (c)

46. **Stats. Question ....**

Correct option is ()

47. For any $b \in \mathbb{R}$, let $S(b)$ denote the set of all broken extremals with one corner of the variational problem:

Minimize $J[y] = \int_0^1 ((y')^4 - 3(y')^2)\, dx,$

subject to $y(0) = 0,\ y(1) = b.$

Then which of the following statements are true ?

(a) $S(2)$ has exactly two elements.

(b) $S\left(\frac{1}{2}\right)$ has exactly one element.

(c) $S(2)$ is empty.

(d) $S\left(\frac{1}{2}\right)$ has exactly two elements.

Correct options are (c), (d)

48. **Stats. Question ....**

Correct option is ()

49. **Stats. Question ....**

Correct option is ()

50. Let $M_2(\mathbb{R})$ denote the $\mathbb{R}$ - vector space of 2×2 matrices with real entries. Let

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} -1 & 0 \\ 1 & 5 \end{pmatrix}$$

Define a linear transformation $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by $T(X) = AXB^t$, where B^t denotes the transpose of the matrix B. Which of the following statements are true ?

(a) $\det(T) = 225$ (b) $\det(T) = -225$ (c) $\text{Trace}(T) = 16$ (d) $\text{Trace}(T) = -16$

Correct options are (a), (c)

51. **Stats. Question ....**

Correct option is ()

52. **Stats. Question ....**

Correct option is ()

53. **Stats. Question ....**

Correct option is ()

54. Which of the following statements are true ?

(a) Let $x, y \in \mathbb{R}$ with $x < y$. Then there exists $r \in \mathbb{Q}$ such that $x < \frac{2^{2024} r}{e} < y$.

(b) Let $(a_n)_{n \geq 2}$ be a sequence of positive real numbers. If there exists a positive real number L such that

$$\limsup_{n \to \infty} \frac{a_n}{\log n} = L, \text{ then } \limsup_{n \to \infty} a_n < \infty.$$

(c) The set of all finite subsets of $\mathbb{Q}$ is countably infinite.

(d) The set of continuous functions from $\mathbb{R}$ to the set $\{0, 1\}$ is infinite.

Correct options are (a), (c)

55. Let $f:[0,1]\to\mathbb{R}$ be a monotonic function. Which of the following statements are true ?

(a) f is Riemann integrable on [0, 1]

(b) The set of discontinuities of f cannot a non-empty open set.

(c) f is a Lebesgue measurable function.

(d) f is a Borel measurable function.

Correct options are (a), (b), (c), (d)

56. **Stats. Question ....**

Correct option is ()

57. Which of the following statements are true ?

(a) The value of the Euler ϕ-function is even for all integers $n\geq 3$.

(b) Let G be a finite group and S a subset of G with $|S|>\frac{|G|}{2}$. Then $\{ab:a,b\in S\}=G$.

(c) The polynomial ring $\mathbb{R}[x_1,...,x_n]$ is a Euclidean domain for all integers $n\geq 1$.

(d) The subset $\left\{f\in C([0,1]):f\left(\frac{1}{2}\right)=0\right\}$ of the ring $C([0,1])$ of continuous functions from [0, 1] to $\mathbb{R}$ is a prime ideal.

Correct options are (a), (b), (d)

58. Let $f:\mathbb{C}\to\mathbb{C}$ be an entire function such that $f(z)=f(iz)$ for all $z\in\mathbb{C}$. Which of the following statements are true ?

(a) $f(z)=f(-z)$ for all $z\in\mathbb{C}$.

(b) $f'(0)=f''(0)=f'''(0)=0$.

(c) There is an entire function $g:\mathbb{C}\to\mathbb{C}$ such that $f(z)=g(z^4)$ for all $z\in\mathbb{C}$.

(d) f is necessarily a constant function.

Correct options are (a), (b), (c)

59. Let $u=u(x,y)$ be the solution of the boundary value problem:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ (x,y)\in(0,1)\times(0,1),$$

$$u(x,0)=e^{\pi x},\ u(x,1)=-e^{\pi x},\ x\in[0,1],$$

$$u(0,y)=\cos(\pi y)+\sin(\pi y),\ y\in[0,1],$$

$$u(1,y)=e^{\pi}(\cos(\pi y)+\sin(\pi y)),\ y\in[0,1].$$

Then there exists a point $(x_0,y_0)\in(0,1)\times(0,1)$ such that

(a) $u(x_0,y_0)=\sqrt{2}\,e^{\pi}$ (b) $u(x_0,y_0)=e^{\pi}$ (c) $u(x_0,y_0)=-1$ (d) $u(x_0,y_0)=-e^{\pi}$

Correct options are (b), (c)

60. **Stats. Question ....**

Correct option is ()