## CSIR-NET – MATHEMATICAL SCIENCES
## JUNE 2018

### PART - B (Mathematical Sciences)

**UNIT-1**

21. Given that there are real constants $a, b, c, d$ such that the identity $\lambda x^2 + 2xy + y^2 = (ax+by)^2 + (cx+dy)^2$ holds for all $x, y \in \mathbb{R}$. This implies

(a) $\lambda = -5$ (b) $\lambda \geq 1$ (c) $0 < \lambda < 1$ (d) there is no such $\lambda \in \mathbb{R}$

22. Let $\mathbb{R}^n$, $n \geq 2$, be equipped with standard inner product. Let $\{v_1, v_2, ..., v_n\}$ be $n$ columns vectors forming an orthonormal basis of $\mathbb{R}^n$. Let $A$ be the $n \times n$ matrix formed by the column vectors $v_1, ..., v_n$. Then,

(a) $A = A^{-1}$ (b) $A = A^T$ (c) $A^{-1} = A^T$ (d) $\det(A) = 1$

23. Given $\{a_n\}, \{b_n\}$ two monotone sequences of real numbers and that $\sum a_n b_n$ is convergent, which of the following is true ?

(a) $\sum a_n$ is convergent and $\sum b_n$ is convergent.

(b) At least one of $\sum a_n, \sum b_n$ is convergent.

(c) $\{a_n\}$ is bounded and $\{b_n\}$ is bounded.

(d) At least one of $\{a_n\}$, $\{b_n\}$ is bounded.

24. Let $S = \left\{(x, y) \mid x^2 + y^2 = \frac{1}{n^2}, \text{ where } n \in \mathbb{N} \text{ and either } x \in \mathbb{Q} \text{ or } y \in \mathbb{Q}\right\}$

Here, $\mathbb{Q}$ is the set of rational numbers and $\mathbb{N}$ is the set of positive integers. Which of the following is true ?

(a) $S$ is a finite non empty set (b) $S$ is countable
(c) $S$ is uncountable (d) $S$ is empty

25. Define the sequence $\{a_n\}$ as follows:

$$a_1 = 1 \text{ and for } n \geq 1,\ a_{n+1} = (-1)^n \left(\frac{1}{2}\right)\left(|a_n| + \frac{2}{|a_n|}\right)$$

Which of the following is true ?

(a) $\limsup a_n = \sqrt{2}$ (b) $\liminf a_n = -\infty$ (c) $\lim a_n = \sqrt{2}$ (d) $\sup a_n = \sqrt{2}$

26. If $\{x_n\}$ is a convergent sequence in $\mathbb{R}$ and $\{y_n\}$ is a bounded sequence in $\mathbb{R}$, then we can conclude that

(a) $\{x_n + y_n\}$ is convergent
(b) $\{x_n + y_n\}$ is bounded
(c) $\{x_n + y_n\}$ has no convergent subsequence
(d) $\{x_n + y_n\}$ has no bounded subsequence

27. The difference $\log(2) - \sum_{n=1}^{100} \frac{1}{2^n \cdot n}$ is :

(a) less than 0
(b) greater than 1
(c) less than $\frac{1}{2^{100} \cdot 101}$
(d) greater than $\frac{1}{2^{100} \cdot 101}$

28. Let $f(x, y) = \log\left(\cos^2(e^{x^2})\right) + \sin(x+y)$. Then, $\frac{\partial}{\partial y}\frac{\partial}{\partial x} f(x, y)$ is :

(a) $\frac{\cos(e^{x^2}) - 1}{1 + \sin^2(e^{x^2})} - \cos(x+y)$
(b) $0$
(c) $-\sin(x+y)$
(d) $\cos(x+y)$

29. Let $A$ be a $(m \times n)$ matrix and $B$ be a $(n \times m)$ matrix over real numbers with $m < n$. Then,

(a) $AB$ is always non-singular.
(b) $AB$ is always singular.
(c) $BA$ is always non-singular.
(d) $BA$ is always singular.

30. If $A$ is a $(2 \times 2)$ matrix over $\mathbb{R}$ with $\det(A + I) = 1 + \det(A)$, then we can conclude that

(a) $\det(A) = 0$
(b) $A = 0$
(c) $\mathrm{Tr}(A) = 0$
(d) $A$ is non-singular

31. The system of equations:

$$1 \cdot x + 2 \cdot x^2 + 3 \cdot xy + 0 \cdot y = 6$$

$$2 \cdot x + 1 \cdot x^2 + 3 \cdot xy + 1 \cdot y = 5$$

$$1 \cdot x - 1 \cdot x^2 + 0 \cdot xy + 1 \cdot y = 7$$

(a) has solutions in rational numbers.
(b) has solutions in real numbers.
(c) has solutions in complex numbers.
(d) has no solutions.

32. The trace of the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}^{20}$ is :

(a) $7^{20}$
(b) $2^{20} + 3^{20}$
(c) $2 \cdot 2^{20} + 3^{20}$
(d) $2^{20} + 3^{20} + 1$

**UNIT-2**

33. Let $f(x) = x^5 - 5x + 2$. Then,

(a) $f$ has no real root
(b) $f$ has exactly one real root
(c) $f$ has exactly three real roots
(d) all roots of $f$ are real

34. Consider the space $S=\{(\alpha,\beta)\mid\alpha,\beta\in\mathbb{Q}\}\subset\mathbb{R}^2$, where $\mathbb{Q}$ is the set of rational numbers. Then,

(a) $S$ is connected in $\mathbb{R}^2$
(b) $S^C$ is connected in $\mathbb{R}^2$
(c) $S$ is closed in $\mathbb{R}^2$
(d) $S^C$ is closed in $\mathbb{R}^2$

35. Suppose that $f$ s a non-constant analytic function defined over $\mathbb{C}$. Then which one of the following is false?

(a) $f$ is unbounded
(b) $f$ sends open sets into open sets.
(c) There exists an open connected domain $U$ on which $f$ is never zero but $\left|f_{|U}\right|$ attains its minimum at some point of $U$.
(d) The image of $f$ is dense in $\mathbb{C}$.

36. The value of the integral $\oint\limits_{|1-z|=1}\frac{e^z}{z^2-1}dz$ is:

(a) 0
(b) $(\pi i)e$
(c) $(\pi i)e-(\pi i)e^{-1}$
(d) $(e+e^{-1})$

37. Let $f:\{z\mid |z|<1\}\to\mathbb{C}$ be a non-constant analytic function. Which of the following conditions can possibly be satisfied by $f$ ?

(a) $f\left(\frac{1}{n}\right)=f\left(\frac{-1}{n}\right)=\frac{1}{n^2}\ \forall\, n\in\mathbb{N}$
(b) $f\left(\frac{1}{n}\right)=f\left(\frac{-1}{n}\right)=\frac{1}{2n+1}\ \forall\, n\in\mathbb{N}$
(c) $\left|f\left(\frac{1}{n}\right)\right|<2^{-n}\ \forall\, n\in\mathbb{N}$
(d) $\frac{1}{\sqrt{n}}<\left|f\left(\frac{1}{n}\right)\right|<\frac{2}{\sqrt{n}}\ \forall\, n\in\mathbb{N}$

38. Consider the map $\phi:\mathbb{C}\setminus\{1\}\to\mathbb{C}$ given by $\phi(z)=\frac{1+z}{1-z}$. Which of the following is true ?

(a) $\phi(\{z\in\mathbb{C}\mid |z|<1\})\subseteq\{z\in\mathbb{C}\mid |z|<1\}$
(b) $\phi(\{z\in\mathbb{C}\mid \mathrm{Re}(z)<0\})\subseteq\{z\in\mathbb{C}\mid \mathrm{Re}(z)<0\}$
(c) $\phi$ is onto
(d) $\phi(\mathbb{C}\setminus\{1\})=\mathbb{C}\setminus\{-1\}$

39. Let $S_7$ denote the group of permutations of the set $\{1, 2, 3, 4, 5, 6, 7\}$. Which of the following is true ?

(a) There are no elements of order 6 in $S_7$.
(b) There are no elements of order 7 in $S_7$.
(c) There are no elements of order 8 in $S_7$.
(d) There are no elements of order 10 in $S_7$.

40. The number of group homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is :

(a) zero
(b) one
(c) five
(d) ten

**UNIT-3**

41. The resolvent kernel for the integral equation $\phi(x)=x^2+\int_0^x e^{t-x}\phi(t)\,dt$ is :

(a) $e^{t-x}$
(b) 1
(c) $e^{x-t}$
(d) $x^2+e^{x-t}$

42. Given that the Lagrangian for the motion of a simple pendulum is :

$$L = \frac{1}{2} ml^2 \dot{\theta}^2 + mgl \cos \theta$$

where $m$ is the mass of the pendulum bob suspended by a string of length $l$, $g$ is the acceleration due to gravity and $\theta$ is the amplitude of the pendulum from the mean position, then a Hamiltonian corresponding to $L$ is :

(a) $H(p, \theta) = \frac{p^2}{2ml^2} + mgl \cos \theta$ (b) $H(p, \theta) = \frac{p^2}{2ml^2} - mgl \cos \theta$

(c) $H(p, \theta) = \frac{p^2}{ml^2} - mgl \cos \theta$ (d) $H(p, \theta) = \frac{3p^2}{2ml^2} + mgl \cos \theta$

43. Consider the ordinary differential equation $y' = y(y-1)(y-2)$. Which of the following statements is true ?

(a) If $y(0) = 0.5$ then $y$ is decreasing. (b) If $y(0) = 1.2$ then $y$ is increasing.

(c) If $y(0) = 2.5$ then $y$ is unbounded. (d) If $y(0) < 0$ then $y$ is bounded below.

44. Consider the ordinary differential equation $y'' + P(x)y' + Q(x)y = 0$, where $P$ and $Q$ are smooth functions. Let $y_1$ and $y_2$ be any two solutions of the ODE. Let $W(x)$ be the corresponding Wronskian. Then which of the following is always true ?

(a) If $y_1$ and $y_2$ are linearly dependent then $\exists x_1, x_2$ such that $W(x_1) = 0$ and $W(x_2) \neq 0$.

(b) If $y_1$ and $y_2$ are linearly independent then $W(x) = 0 \ \forall \ x$.

(c) If $y_1$ and $y_2$ are linearly dependent then $W(x) \neq 0 \ \forall \ x$.

(d) If $y_1$ and $y_2$ are linearly independent then $W(x) \neq 0 \ \forall \ x$.

45. The Cauchy problem $2u_x + 3u_y = 5,\ u = 1$ on the line $3x - 2y = 0$ has :

(a) exactly one solution (b) exactly two solutions

(c) infinitely many solutions (d) no solution

46. Let $u$ be the unique solution of

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0,\ x \in \mathbb{R},\ t > 0$$

$$u(x, 0) = f(x),\ \frac{\partial u}{\partial t}(x, 0) = 0,\ x \in \mathbb{R}$$

where $f : \mathbb{R} \to \mathbb{R}$ satisfies the relations $f(x) = x(1-x) \ \forall \ x \in [0,1]$ and $f(x+1) = f(x) \ \forall \ x \in \mathbb{R}$. Then $u\left(\frac{1}{2}, \frac{5}{4}\right)$ is :

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{3}{16}$ (d) $\frac{5}{16}$

47. The values of a, b, c such that $\int_0^h f(x)\,dx = h\left\{af(0)+bf\left(\frac{h}{3}\right)+cf(h)\right\}$ is exact for polynomials $f$ of degree as high as possible are:

(a) $a=0,\ b=\frac{3}{4},\ c=\frac{1}{4}$ (b) $a=\frac{3}{4},\ b=\frac{2}{4},\ c=\frac{1}{4}$

(c) $a=\frac{-2}{4},\ b=\frac{3}{4},\ c=\frac{1}{4}$ (d) $a=0,\ b=\frac{1}{4},\ c=\frac{3}{4}$

48. Consider $J[y]=\int_0^1\left[(y')^2+2y\right]dx$ subject to $y(0)=0,\ y(1)=1$. Then $\inf J[y]$ is :

(a) $\frac{23}{12}$ (b) $\frac{21}{24}$ (c) $\frac{18}{25}$ (d) does not exist

**UNIT-4**

49. In a Latin Square Design the "error degrees of freedom" is 30. The "treatment degrees of freedom" for any treatment is :

(a) 4 (b) 5 (c) 6 (d) 7

50. Suppose that $|3x|+|2y|\le 1$. Then the maximum value of $9x+4y$ is :

(a) 1 (b) 2 (c) 3 (d) 4

51. A standard fair die is rolled until some face other than 5 or 6 turn up. Let $X$ denote the face value of the last roll, and $A=\{X \text{ is even}\}$ and $B=\{X \text{ is at most } 2\}$. Then,

(a) $P(A\cap B)=0$ (b) $P(A\cap B)=\frac{1}{6}$ (c) $P(A\cap B)=\frac{1}{4}$ (d) $P(A\cap B)=\frac{1}{3}$

52. Let $X$ and $Y$ be i.i.d. uniform (0, 1) random variables. Let $Z=\max(X,Y)$ and $W=\min(X,Y)$. Then $P\left((Z-W)>\frac{1}{2}\right)$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

53. Consider a Markov chain having state space $S=\{1, 2, 3, 4\}$ with transition probability matrix $P=(p_{ij})$ given by :

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1/2 & 0 & 1/2 & 0 \\ 2 & 1/4 & 1/4 & 1/4 & 1/4 \\ 3 & 1/3 & 0 & 1/3 & 1/3 \\ 4 & 1/2 & 0 & 1/2 & 0 \end{array}$$

Then,

(a) $\lim_{n\to\infty} p_{2,2}^{(n)} = 0, \sum_{n=0}^{\infty} p_{2,2}^{(n)} = \infty$

(b) $\lim_{n\to\infty} p_{2,2}^{(n)} = 0, \sum_{n=0}^{\infty} p_{2,2}^{(n)} < \infty$

(c) $\lim_{n\to\infty} p_{2,2}^{(n)} = 1, \sum_{n=0}^{\infty} p_{2,2}^{(n)} = \infty$

(d) $\lim_{n\to\infty} p_{2,2}^{(n)} = 1, \sum_{n=0}^{\infty} p_{2,2}^{(n)} < \infty$

54. Let $X_1, X_2, X_3$ be i.i.d. standard normal variables. Which of the following is true ?

(a) $\dfrac{\sqrt{2}[X_1]}{\sqrt{X_2^2 + X_3^2}} \sim t_2$

(b) $\dfrac{X_1 - 2X_2 + X_3}{\sqrt{2}\left|X_1 + X_2 + X_3\right|} \sim t_1$

(c) $\dfrac{(X_1 - X_2)^2}{(X_1 + X_2)^2} \sim F_{2,2}$

(d) $\dfrac{3X_1^2}{X_1^2 + X_2^2 + X_3^2} \sim F_{1,3}$

55. Suppse that the lifetime of an electric bulb follows an exponential distribution with mean $\theta$ hours. In order to estimate $\theta, n$ bulbs are switched on at the same time. After $t$ hours, $n - m(>0)$ bulbs are found to be in functioning state. If the lifetimes of the other $m(>0)$ bulbs are noted as $x_1, x_2, ..., x_m$, respectively, then the maximum likelihood estimate of $\theta$ is given by:

(a) $\hat{\theta} = \dfrac{t}{\log \dfrac{n}{n-m}}$

(b) $\hat{\theta} = \dfrac{\sum_{i=1}^{m} x_1}{m}$

(c) $\hat{\theta} = \dfrac{\sum_{i=1}^{m} x_1 + (n-m)t}{n}$

(d) $\hat{\theta} = \dfrac{\sum_{i=1}^{m} x_1 + (n-m)t}{m}$

56. Let $X_1, X_2, ...., X_n$ be i.i.d. uniform $(\theta_1, \theta_2)$ variables, where $\theta_1 < \theta_2$ are unknown parameters. Which of the following is an ancillary statistic ?

(a) $\dfrac{X_{(k)}}{X_{(n)}}$ for any $k < n$

(b) $\dfrac{X_{(n)} - X_{(k)}}{X_{(n)}}$ for any $k > n$

(c) $\dfrac{X_{(k)}}{X_{(n)} - X_{(k)}}$ for any $k < n$

(d) $\dfrac{X_{(k)} - X_{(1)}}{X_{(n)} - X_{(k)}}$ for any $k$ where $1 < k < n$

57. Consider the problem of estimation of a parameter $\theta$ on the basis of $X$, where $X \sim N(\theta, 1)$ and $-\infty < \theta < \infty$. Under squared error loss, $X$ has uniformly smaller risk than that of $kX$, for

(a) $k < 0$ (b) $0 < k < 1$ (c) $k > 1$ (d) no value of $k$

58. To test the equality of effects of 10 schools against all alternatives, we take a random sample of 5 students from each school and note their marks in a common examination. "Between sum of squares" and "total sum of squares" are found to be 180 and 500 respectively. What is the $p$-value for the standard $F$-test ?

(a) $P\left[F_{4,45} > 1.5\right]$ (b) $P\left[F_{9,40} \geq 1.6\right]$ (c) $P\left[F_{4,45} \geq 3.6\right]$ (d) $P\left[F_{9,40} \geq 2.5\right]$

59. The covariance matrix of a four dimensional random vector $X$ is of the form:

$$\begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{bmatrix}; \text{ where } \rho < 0.$$

If $v$ is the variance of the first principal component, then

(a) $v$ cannot exceed 5/4
(b) $v$ can exceed 5/4, but cannot exceed 4/3
(c) $v$ can exceed 4/3, but cannot exceed 3/2
(d) $v$ can exceed 3/2

60. A simple random sample of size $n$ will be drawn from a class of 125 students, and the mean mathematics score of the sample will be computed. If the standard error of the sample mean for "with replacement sampling" is twice as much as the standard error of the sample mean for "without replacement" sampling the value of $n$ is:

(a) 32
(b) 63
(c) 79
(d) 94

## PART - C (Mathematical Sciences)

**UNIT-1**

61. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ and define for $x, y, z \in \mathbb{R}$; $Q(x, y, z) = (x\ y\ z)\ A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Which of the following statements are true ?

(a) The matrix of second order partial derivatives of the quadratic form $Q$ is $2A$.
(b) The rank of the quadratic form $Q$ is 2.
(c) The signature of the quadratic for $Q$ is $(+\,+\,0)$.
(d) The quadratic form $Q$ takes the value 0 for some non-zero vector $(x, y, z)$.

62. For each $\alpha \in \mathbb{R}$, let $S_\alpha = \left\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = \alpha^2\right\}$. Let $E = \bigcup_{\alpha \in \mathbb{R} \setminus \mathbb{Q}} S_\alpha$. Which of the following are true ?

(a) The Lebesgue measure of $E$ is infinite.
(b) $E$ contains a non-empty open set.
(c) $E$ is path connected.
(d) Every open set containing $E^C$ has infinite Lebesgue measure.

63. Which of the following sets are uncountable ?

(a) The set of all functions from $\mathbb{R}$ to $\{0, 1\}$
(b) The set of all functions from $\mathbb{N}$ to $\{0, 1\}$
(c) The set of all finite subset of $\mathbb{N}$
(d) The set of all subset of $\mathbb{N}$

64. Let $A = \left\{ t \sin\left(\frac{1}{t}\right) \mid t \in \left(0, \frac{2}{\pi}\right) \right\}$. Which of the following statements are true ?

(a) $\sup(A) < \frac{2}{\pi} + \frac{1}{n\pi}$ for all $n \geq 1$
(b) $\inf(A) > \frac{-2}{3\pi} - \frac{1}{n\pi}$ for all $n \geq 1$
(c) $\sup(A) = 1$
(d) $\inf(A) = -1$

65. Let $C_C(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \mid f$ is continuous and there exists a compact set $K$ such that $f(x) = 0$ for all $x \in K^C\}$. Let $g(x) = e^{-x^2}$ for all $x \in \mathbb{R}$. Which of the following statements are true ?

(a) There exists a sequence $\{f_n\}$ in $C_C(\mathbb{R})$ such that $f_n \to g$ uniformly.

(b) There exists a sequence $\{f_n\}$ in $C_C(\mathbb{R})$ such that $f_n \to g$ pointwise.

(c) If a sequence in $C_C(\mathbb{R})$ converges pointwise to $g$ then it must converge uniformly to $g$.

(d) There does not exist any sequence in $C_C(\mathbb{R})$ converging pointwise to $g$.

66. Given that :

$$a(n) = \frac{1}{10^{100}} 2^n$$
$$b(n) = 10^{100} \log(n)$$
$$c(n) = \frac{1}{10^{10}} n^2$$

Which of the following statements are true ?

(a) $a(n) > c(n)$ for all sufficiently large $n$.

(b) $b(n) > c(n)$ for all sufficiently large $n$.

(c) $b(n) > n$ for all sufficiently large $n$.

(d) $a(n) > b(n)$ for all sufficiently large $n$.

67. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \dfrac{a}{1 + bx^2}$, $a, b \in \mathbb{R};\ b \geq 0$. Which of the following are true ?

(a) $f$ is uniformly continuous on compact intervals of $\mathbb{R}$ for all values of $a$ and $b$.

(b) $f$ is uniformly continuous on $\mathbb{R}$ and is bounded for all values of $a$ and $b$.

(c) $f$ is uniformly continuous on $\mathbb{R}$ only if $b = 0$.

(d) $f$ is uniformly continuous on $\mathbb{R}$ and unbounded if $a \neq 0, b \neq 0$.



68. Let $\alpha = \displaystyle\int_0^\infty \frac{1}{1+t^2} dt$. Which of the following are true ?

(a) $\dfrac{d\alpha}{dt} = \dfrac{1}{1+t^2}$

(b) $\alpha$ is a rational number

(c) $\log(\alpha) = 1$

(d) $\sin(\alpha) = 1$

69. Which of the following functions are of bounded variation ?

(a) $x^2 + x + 1$ for $x \in (-1, 1)$

(b) $\tan\left(\dfrac{nx}{2}\right)$ for $x \in (-1, 1)$

(c) $\sin\left(\dfrac{x}{2}\right)$ for $x \in (-\pi, \pi)$

(d) $\sqrt{1 - x^2}$ for $x \in (-1, 1)$

70. Let $M_n(\mathbb{R})$ denote the space of all $n \times n$ real matrices identified with the Euclidean space $\mathbb{R}^{n^2}$. Fix a column vector $x \neq 0$ in $\mathbb{R}^n$. Define $f : M_n(\mathbb{R}) \to \mathbb{R}$ by $f(A) = \langle A^2 x, x \rangle$. Then,

(a) $f$ is linear
(b) $f$ is differentiable
(c) $f$ is continuous but not differentiable
(d) $f$ is unbounded

71. For any $y \in \mathbb{R}$, let $[y]$ denote the greatest integer less than or equal to $y$.

(a) $f$ is continuous on $\mathbb{R}^2$
(b) For every $y \in \mathbb{R},\ x \mapsto f(x, y)$ is continuous on $\mathbb{R} \backslash \{0\}$
(c) For every $x \in \mathbb{R},\ y \mapsto f(x, y)$ is continuous on $\mathbb{R}$
(d) $f$ is continuous at no point of $\mathbb{R}^2$

72. Let $V$ denote the vector space of all sequence $a = (a_1, a_2, ...)$ of real numbers such that $\sum 2^n |a_n|$ converges. Define $\|\bullet\| : V \to \mathbb{R}$ by $\|a\| = \sum 2^n |a_n|$. Which of the following are true ?

(a) $V$ contains only the sequence (0, 0, ...)
(b) $V$ is finite dimensional
(c) $V$ has a countable linear basis
(d) $V$ is a complete normed space

73. Let $V$ be a vector space over $\mathbb{C}$ with dimension $n$. Let $T : V \to V$ be a linear transformation with only 1 as eigenvalue. Then which of the following must be true ?

(a) $T - I = 0$
(b) $(T - I)^{n-1} = 0$
(c) $(T - I)^n = 0$
(d) $(T - I)^{2n} = 0$

74. If $A$ is a $(5 \times 5)$ matrix and the dimension of the solution space of $Ax = 0$ is at least two, then

(a) $\text{Rank}(A^2) \leq 3$
(b) $\text{Rank}(A^2) \geq 3$
(c) $\text{Rank}(A^2) = 3$
(d) $\text{Det}(A^2) = 0$

75. Let $A \in M_3(\mathbb{R})$ be such that $A^8 = I_{3\times3}$. Then,

(a) minimal polynomial of $A$ can only be of degree 2.
(b) minimal polynomial of $A$ can only be of degree 3.
(c) either $A = I_{3\times3}$ or $A = -I_{3\times3}$.
(d) there are uncountably many $A$ satisfying the above.

76. Let $A$ be an $n \times n$ matrix (with $n > 1$) satisfying $A^2 - 7A + 12 I_{n\times n} = O_{n\times n}$, where $I_{n\times n}$ and $O_{n\times n}$ denote the identity matrix and zero matrix of order $n$ respectively. Then which of the following statements are true ?

(a) $A$ is invertible
(b) $t^2 - 7t + 12n = 0$, where $t = Tr(A)$
(c) $d^2 - 7d + 12 = 0$, where $d = \det(A)$
(d) $\lambda^2 - 7\lambda + 12 = 0$, where $\lambda$ is an eigenvalue of $A$

77. Let $A$ be a $(6\times 6)$ matrix over $\mathbb{R}$ with characteristic polynomial $=(x-3)^2(x-2)^4$ and minimal polynomial $=(x-3)(x-2)^2$. Then Jordan canonical form of $A$ can be :

(a) $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

78. Let $V$ be an inner product space and $S$ be a subset of $V$. Let $\overline{S}$ denote the closure of $S$ in $V$ with respect to the topology induced by the metric given by the inner product. Which of the following statements are true ?

(a) $S=(S^{\perp})^{\perp}$ (b) $\overline{S}=(S^{\perp})^{\perp}$ (c) $\overline{span(S)}=(S^{\perp})^{\perp}$ (d) $S^{\perp}=((S^{\perp})^{\perp})^{\perp}$

**UNIT-2**

79. Let $G=S_3$ be the permutation group of 3 symbols. Then,

(a) $G$ is isomorphic to a subgroup of a cyclic group.
(b) there exists a cyclic group $H$ such that $G$ maps homomorphically onto $H$.
(c) $G$ is a product of cyclic group.
(d) there exists a nontrivial group homomorphism from $G$ to the additive group $(\mathbb{Q},+)$ of rational numbers.

80. Let $S$ be the set of polynomials $f(x)$ with integer coefficients satisfying

$f(x)\equiv 1 \bmod (x-1)$

$f(x)\equiv 0 \bmod (x-3)$

Which of the following statements are true ?

(a) $S$ is empty (b) $S$ is a singleton
(c) $S$ is a finite non-empty set (d) $S$ is countably infinite

81. Let $\Omega$ be an open connected subset of $\mathbb{C}$. Let $E=\{z_1, z_2, ..., z_r\}\subseteq\Omega$. Suppose that $f:\Omega\to\mathbb{C}$ is a function such that $f|_{(\Omega\setminus E)}$ is analytic. Then $f$ is analytic on $\Omega$ if

(a) $f$ is continuous on $\Omega$.

(b) $f$ is bounded on $\Omega$.

(c) For every $j$, if $\sum_{m\in\mathbb{Z}} a_m(z-z_j)^m$ is Laurent series expansion of $f$ at $z_j$, then $a_m=0$ for $m=-1,-2,-3,...$

(d) For every $j$, if $\sum_{m\in\mathbb{Z}} a_m(z-z_j)^m$ is Laurent series expansion of $f$ at $z_j$, then $a_{-1}=0$.

82. Suppose that $f:\mathbb{C}\to\mathbb{C}$ is an analytic function. Then $f$ is a polynomial if

(a) for any point $a\in\mathbb{C}$, if $f(z)=\sum_0^\infty a_n(z-a)^n$ is a power series expansion at $a$, then $a_n=0$ for at least one $n$.

(b) $\lim_{|z|\to\infty}|f(z)|=\infty$

(c) $\lim_{|z|\to\infty}|f(z)|=M$ for some $M$

(d) $|f(z)|\le M|z|^n$ for $|z|$ sufficiently large and for some $n$.

83. Let $\mathbb{D}$ be the open unit disk centered at 0 in $\mathbb{C}$ and $f:\mathbb{D}\to\mathbb{C}$ be an analytic function. Let $f=u+iv$, where $u, v$ are the real and imaginary parts of $f$. If $f(z)=\sum a_n z^n$ is the power series of $f$, then $f$ is constant if

(a) $f$ is analytic

(b) $u\left(\frac{1}{2}\right)\ge u(z),\ \forall\ z\in\mathbb{D}$

(c) The set $\{n\in\mathbb{N}\mid a_n=0\}$ is infinite

(d) For any closed curve $\gamma$ in $\mathbb{D}$, $\int_\gamma \frac{f(z)\,dz}{(z-2)^2}=0\ \forall\ a\in\mathbb{D}$ with $|a|\ge\frac{1}{2}$

84. Which of the following statements are true ?

(a) If $\{a_k\}$ is bounded then $\sum_0^\infty a_k z^k$ defines an analytic function on the open unit disk.

(b) If $\sum_0^\infty a_k z^k$ defines an analytic function on the open unit disk then $\{a_k\}$ must converge to zero.

(c) If $f(z)=\sum_0^\infty a_k z^k$ and $g(z)=\sum_0^\infty b_k z^k$ are two power series functions whose radii of convergence are 1, then the product $f\cdot g$ has a power series representation of the form $\sum_0^\infty c_k z^k$ on the open unit disk.

(d) If $f(z)=\sum_0^\infty a_k z^k$ has a radius of convergence 1, then $f$ is continuous on $\Omega=\{z\in\mathbb{C}\,\big|\,|z|\le 1\}$.

85. Which of the following statements are true ?

(a) Every compact metric space is separable.

(b) If a metric space $(X, d)$ is separable, then the metric $d$ is not the discrete metric.

(c) Every separable metric space is second countable.

(d) Every first countable topological space is separable.

86. Let $X$ be a topological space and $A$ be a non-empty subset of $X$. Then one can conclude that —

(a) $A$ is dense in $X$, if $(X \setminus A)$ is nowhere dense in $X$.

(b) $(X \setminus A)$ is nowhere dense in $X$, if $A$ is dense in $X$.

(c) $A$ is dense in $X$, if the interior of $(X \setminus A)$ is empty.

(d) The interior of $(X \setminus A)$ is empty, if $A$ is dense in $X$.

87. Which of the following statements are true ?
(a) The multiplicative group of a finite field is always cyclic.
(b) The additive group of a finite field is always cyclic.
(c) There exists a finite field of any given order.
(d) There exists at most one finite field (upto isomorphism) of any given order.

88. Let $f(x) \in \mathbb{Z}[x]$ be a monic polynomial. Then the roots of $f$

(a) can belong to $\mathbb{Z}$.
(b) always belong to $(\mathbb{R} \setminus \mathbb{Q}) \cup \mathbb{Z}$.
(c) always belong to $(\mathbb{C} \setminus \mathbb{Q}) \cup \mathbb{Z}$.
(d) can belong to $(\mathbb{Q} \setminus \mathbb{Z})$.

89. Which of the following statements are true ?
(a) A subring of an integral domain is an integral domain.
(b) A subring of a unique factorization domain (UFD) is a UFD.
(c) A subring of a principal ideal domain (PID) is a PID.
(d) A subring of an Euclidean domain is an Euclidean domain.

90. Let $G$ be a group with $|G| = 96$. Suppose $H$ and $K$ are subgroups of $G$ with $|H| = 12$ and $|K| = 16$. Then,

(a) $H \cap K = \{e\}$
(b) $H \cap K \neq \{e\}$
(c) $H \cap K$ is Abelian
(d) $H \cap K$ is not Abelian

**UNIT-3**

91. Assume that a non-singular matrix $A = L + D + U$, where $L$ and $U$ are lower and upper triangular matrices respectively with all diagonal entries are zero, and $D$ is a diagonal matrix. Let $x^*$ be the solution of $Ax = b$. Then the Gauss-Seidel iteration method $x^{(k+1)} = Hx^{(k)} + c,\ k = 0, 1, 2, \ldots$ with $\|H\| < 1$ converges $x^*$ provided $H$ is equal to :

(a) $-D^{-1}(L+U)$
(b) $-(D+L)^{-1}U$
(c) $-D(L+U)^{-1}$
(d) $-(L-D)^{-1}U$

92. Let $a$ be a fixed real constant. Consider the first order partial differential equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0,\ x \in \mathbb{R},\ t > 0$$

with the initial data $u(x, 0) = u_0(x),\ x \in \mathbb{R}$ where $u_0$ is a continuously differentiable function. Consider the following two statements.

$S_1$: There exists a bounded function $u_0$ for which the solution $u$ is unbounded.

$S_2$: If $u_0$ vanishes outside a compact set then for each fixed $T > 0$ there exists a compact set $K_T \subset \mathbb{R}$ such that $u(x, T)$ vanishes for $x \notin K_T$.

Which of the following are true ?

(a) $S_1$ is true and $S_2$ is false.
(b) $S_1$ is true and $S_2$ is also true.
(c) $S_1$ is false and $S_2$ is true.
(d) $S_1$ is false and $S_2$ is also false.

93. If $u(x, t)$ is the solution of

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$$

$$u(x, 0) = 1 + x + \sin(\pi x)\cos(\pi x)$$

$$u(0, t) = 1,\ u(1, t) = 2$$

Then,

(a) $u\left(\frac{1}{2}, \frac{1}{4}\right) = \frac{3}{2}$
(b) $u\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{3}{2}$
(c) $u\left(\frac{1}{4}, \frac{3}{4}\right) = \frac{5}{4} + \frac{1}{2}e^{-3\pi^2}$
(d) $u\left(\frac{1}{4}, 1\right) = \frac{5}{4} + \frac{1}{2}e^{-4\pi^2}$

94. Assume that $a : [0, \infty) \to \mathbb{R}$ is a continuous function. Consider the ordinary differential equation:

$$y'(x) = a(x)\,y(x),\ x > 0,\ y(0) = y_0 \neq 0$$

Which of the following statements are true ?

(a) If $\int_0^\infty |a(x)|\,dx < \infty$, then $y$ is bounded.
(b) If $\int_0^\infty |a(x)|\,dx < \infty$, then $\lim_{x\to\infty} y(x)$ exists.
(c) If $\lim_{x\to\infty} a(x) = 1$, then $\lim_{x\to\infty} |y(x)| = \infty$.
(d) If $\lim_{x\to\infty} a(x) = 1$, then $y$ is monotone.

95. Consider the system of differential equations:

$$\frac{dx}{dt} = 2x - 7y;\ \&\ \frac{dy}{dt} = 3x - 8y$$

Then the critical point $(0, 0)$ of the system is an:

(a) asymptotically stable node.
(b) unstable node.
(c) asymptotically stable spiral.
(d) unstable spiral.

96. Consider the Sturm-Liouville problem $y'' + \lambda y = 0,\ y(0) = 0$ and $y(\pi) = 0$. Which of the following statements are true ?

(a) There exists only countably many characteristic values.
(b) There exists uncountably many characteristic values.
(c) Each characteristic function corresponding to the characteristic value $\lambda$ has exactly $|\sqrt{\lambda}| - 1$ zeros in $(0, \pi)$.
(d) Each characteristic function corresponding to the characteristic value $\lambda$ has exactly $|\sqrt{\lambda}|$ zeros in $(0, \pi)$.

97. The Hamiltonian for a simple harmonic oscillator is $H(p, q) = \frac{p^2}{2m} + \frac{k}{2}q^2$. Then a possible Lagrangian corresponding to $H$ can be:

(a) $L = \frac{1}{2}m\dot{q}^2 - \frac{k}{2}q^2$ (b) $L = \frac{1}{2}m\dot{q}^2 - \frac{k}{2}(q^2 + 3q^2\dot{q})$

(c) $L = \frac{1}{2}m\dot{q}^2 + \frac{k}{2}q^2$ (d) $L = \frac{1}{2}m\dot{q}^2 + \frac{k}{2}(q^2 + 3q^2\dot{q})$

98. The values of $\lambda$ for which the following equation has a non-trivial solution

$\phi(x) = \lambda\int_0^{\pi} K(x,t)\phi(t)\,dt,\ (0 \le x \le \pi)$; where $K(x,t) = \begin{cases} \sin x \cos t & ; \quad 0 \le x \le t \\ \cos x \sin t & ; \quad t \le x \le \pi \end{cases}$ are:

(a) $\left(n + \frac{1}{2}\right)^2 - 1,\ n \in \mathbb{N}$ (b) $n^2 - 1,\ n \in \mathbb{N}$

(c) $\frac{1}{2}(n+1)^2 - 1,\ n \in \mathbb{N}$ (d) $\frac{1}{2}(2n+1)^2 - 1,\ n \in \mathbb{N}$

99. Consider the integral equations:

$$\phi(x) = \lambda\int_0^{\pi}[\cos x \cos t - 2\sin x \sin t]\phi(t)\,dt + \cos 7x,\ (0 \le x \le \pi)$$

Which of the following statements are true ?

(a) For every $\lambda \in \mathbb{R}$, a solution exists.

(b) There exists $\lambda \in \mathbb{R}$ such that solution does not exist.

(c) There exists $\lambda \in \mathbb{R}$ such that there are more than one but finitely many solutions.

(d) There exists $\lambda \in \mathbb{R}$ such that there are infinitely many solutions.

100. The extremal of the functional $J[y] = \int_0^1 y'^2(x)\,dx$ subject to $y(0) = 0,\ y(1) = 1$ and $\int_0^1 y(x)\,dx = 0$ is :

(a) $3x^2 - 2x$ (b) $8x^3 - 9x^2 + 2x$ (c) $\frac{5}{3}x^4 - \frac{2}{3}x$ (d) $\frac{-21}{2}x^5 + 10x^4 + 4x^3 - \frac{5}{2}x$

101. The admissible extremal for $J[y] = \int_0^{\log 3}\left[e^{-x}y'^2 + 2e^x(y' + y)\right]dx$, where $y(\log 3) = 1$ and $y(0)$ is free is:

(a) $4 - e^x$ (b) $10 - e^{2x}$ (c) $e^x - 2$ (d) $e^{2x} - 8$

102. The forward difference operator is defined as, $\Delta U_n = U_{n+1} - U_n$. Then which of the following difference equations has an unbounded general solution ?

(a) $\Delta^2 U_n - 3\Delta U_n + 2U_n = 0$

(b) $\Delta^2 U_n + \Delta U_n + \frac{1}{4} U_n = 0$

(c) $\Delta^2 U_n - 2\Delta U_n + 2U_n = 0$

(d) $\Delta^2 U_{n+1} - \frac{1}{3}\Delta^2 U_n = 0$

**UNIT-4**

103. STATS QUESTIONS ....................................

## CSIR-NET – MATHEMATICAL SCIENCES
## JUNE 2018

### (Answer Key) — PART - B (Mathematical Sciences)

| | | | | |
|---|---|---|---|---|
| **21.** (b) | **22.** (c) | **23.** (d) | **24.** (b) | **25.** (a) |
| **26.** (b) | **27.** (c) | **28.** (c) | **29.** (d) | **30.** (c) |
| **31.** (d) | **32.** (c) | **33.** (c) | **34.** (b) | **35.** (c) |
| **36.** (b) | **37.** (a) | **38.** (d) | **39.** (c) | **40.** (d) |
| **41.** (b) | **42.** (b) | **43.** (c) | **44.** (d) | **45.** (d) |
| **46.** (c) | **47.** (a) | **48.** (a) | **49.** (c) | **50.** (c) |
| **51.** (c) | **52.** (c) | **53.** (b) | **54.** (b) | **55.** (d) |
| **56.** (d) | **57.** (c) | **58.** (d) | **59.** (b) | **60.** (d) |

### (Answer Key) — PART - C (Mathematical Sciences)

| | | | | |
|---|---|---|---|---|
| **61.** (d) | **62.** (a) | **63.** (a), (b), (d) | **64.** (a), (b) | **65.** (a), (b) |
| **66.** (a), (d) | **67.** (a), (b) | **68.** (d) | **69.** (a), (c), (d) | **70.** (b), (d) |
| **71.** (b) | **72.** (d) | **73.** (c), (d) | **74.** (a), (d) | **75.** (d) |
| **76.** (a), (d) | **77.** (b), (c) | **78.** (c), (d) | **79.** (b) | **80.** (a) |
| **81.** (a), (c) | **82.** (a), (b), (c), (d) | **83.** (a), (b), (d) | **84.** (a), (c) | **85.** (a), (c) |
| **86.** (a), (c), (d) | **87.** (a), (d) | **88.** (a), (c) | **89.** (a) | **90.** (b), (c) |
| **91.** (b) | **92.** (c) | **93.** (a), (b), (c), (d) | **94.** (a), (b), (c) | **95.** (a) |
| **96.** (a), (c) | **97.** (a), (b) | **98.** (a) | **99.** (a), (d) | **100.** (a) |
| **101.** (a) | **102.** (a), (c), (d) | | | |