CSIR-NET – MATHEMATICAL SCIENCES
JUNE 2015

PART - B (Mathematical Sciences)

UNIT-1

21. Let $f : X \to X$ such that $f(f(x)) = x$ for all $x \in X$. Then,

(a) f is one-to-one and onto.
(b) f is one-to-one, but not onto.
(c) f is onto but not one-to-one.
(d) f need not be either one-to-one or onto.

22. Let V be the space of twice differentiable functions on $\mathbb{R}$ satisfying $f'' - 2f' + f = 0$. Define $T : V \to \mathbb{R}^2$ by $T(f) = (f'(0), f(0))$. Then T is :

(a) one-to-one and onto
(b) one-to-one but not onto
(c) onto but not one-to-one
(d) neither one-to-one nor onto

23. The limit $\lim_{x\to 0} \frac{1}{x}\int_x^{2x} e^{-t^2}\,dt$

(a) does not exist
(b) is infinite
(c) exists and equals 1
(d) exists and equals 0

24. The sum of the series: $\frac{1}{1!} + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \cdots$ equals

(a) e
(b) $\frac{e}{2}$
(c) $\frac{3e}{2}$
(d) $1+\frac{e}{2}$

25. Which of the following subsets of $\mathbb{R}^n$ is compact (with respect to the usual topology of $\mathbb{R}^n$)?

(a) $\{(x_1, x_2, ..., x_n) : |x_i| < 1, 1 \le i \le n\}$
(b) $\{(x_1, x_2, ..., x_n) : x_1 + x_2 + \cdots + x_n = 0\}$
(c) $\{(x_1, x_2, ..., x_n) : x_i \ge 0, 1 \le i \le n\}$
(d) $\{(x_1, x_2, ..., x_n) : 1 \le x_i \le 2^i, 1 \le i \le n\}$

26. A polynomial of odd degree with real coefficient must have

(a) at least one real root.
(b) no real root.
(c) only real roots.
(d) at least one root which is not real.

27. Let A, B be $n \times n$ matrices. Which of the following equals trace (A^2B^2)?

(a) $(\text{trace}(AB))^2$
(b) $\text{trace}(AB^2A)$
(c) $\text{trace}((AB)^2)$
(d) $\text{trace}(BABA)$

28. Let A be an $m \times n$ matrix of rank n with real entries. Choose the correct statement.

(a) $Ax = b$ has a solution for any b.
(b) $Ax = 0$ does not have a solution.
(c) If $Ax = b$ has a solution, then it is unique.
(d) $y'A = 0$ for some non-zero y, where y' denotes the transpose of the vector y.

29. The row space of a 20×50 matrix A has dimension 13. What is the dimension of the space of solutions of $Ax = 0$?

(a) 7 (b) 13 (c) 33 (d) 37

30. Let T be a 4×4 real matrix such that $T^4 = 0$. Let $k_i := \dim \ker T^i$ for $1 \le i \le 4$. Which of the following is NOT a possibility for the sequence $k_1 \le k_2 \le k_3 \le k_4$?

(a) $3 \le 4 \le 4 \le 4$ (b) $1 \le 3 \le 4 \le 4$ (c) $2 \le 4 \le 4 \le 4$ (d) $2 \le 3 \le 4 \le 4$

31. Given a 4×4 real matrix A, let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation defined by $Tv = Av$, where we think of $\mathbb{R}^4$ as the set of real 4×1 matrices. For which choices of A given below, do Image (T) and Image (T^2) have respective dimensions 2 and 1 ? [* denotes a non-zero entry]

(a) $A = \begin{bmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} 0 & 0 & * & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & * & 0 \end{bmatrix}$ (d) $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{bmatrix}$

32. Which of the following is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$?

(1) $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x+y \end{pmatrix}$ (2) $g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$ (3) $h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-x \\ x+y \end{pmatrix}$

(a) only f (b) only g

(c) only h (d) all the transformation f, g and h

UNIT-2

33. Let f be a real valued harmonic function on $\mathbb{C}$, that is, f satisfies the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Define the functions:

$$g = \frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y};\ h = \frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}$$

Then,

(a) g and h are both holomorphic functions.
(b) g is holomorphic, but h need not be holomorphic.
(c) h is holomorphic, but g need not be holomorphic.
(d) both g and h are identically equal to the zero function.

34. $\int\limits_{|z+1|=2} \frac{z^2}{4-z^2} dz =$

(a) 0 (b) $-2\pi i$ (c) $2\pi i$ (d) 1

35. Let D be the set of tuples $(w_1, ..., w_{10})$, where $w_i \in \{1, 2, 3\}$, $1 \le i \le 10$ and $w_i + w_{i+1}$ is an even number for each i with $1 \le i \le 9$. Then the number of elements in D is :

(a) $2^{11} + 1$ (b) $2^{10} + 1$ (c) $3^{10} + 1$ (d) $3^{11} + 1$

36. How many elements does the set, $\{z \in \mathbb{C} \mid z^{60} = -1, z^k \ne -1 \text{ for } 0 < k < 60\}$ have ?

(a) 24 (b) 30 (c) 32 (d) 45

37. Up to isomorphism, the number of abelian groups of order 108 is :

(a) 12 (b) 9 (c) 6 (d) 5

38. Let R be the ring $\mathbb{Z}[x]/((x^2 + x + 1)(x^3 + x + 1))$ and I be the ideal generated by 2 in R. What is the cardinality of the ring R ?

(a) 27 (b) 32 (c) 64 (d) Infinite

39. The number of subfields of a field of cardinality 2^{100} is :

(a) 2 (b) 4 (c) 9 (d) 100

40. Let, for each $n \ge 1$, C_n be the open disc in $\mathbb{R}^2$, with centre at the point $(n, 0)$ and radius equal to n. Then, $C = \bigcup\limits_{n \ge 1} C_n$ is:

(a) $\{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } |y| < x\}$ (b) $\{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } |y| < 2x\}$

(c) $\{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } |y| < 3x\}$ (d) $\{(x, y) \in \mathbb{R}^2 : x > 0\}$

UNIT-3

41. Let $a, b \in \mathbb{R}$ be such that $a^2 + b^2 \ne 0$. Then the Cauchy problem

$$a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} = 1;\ x, y \in \mathbb{R}$$

$$u(x, y) = x \text{ on } ax + by = 1$$

(a) has more than one solution if either a or b is zero.
(b) has no solution.
(c) has a unique solution.
(d) has infinitely many solutions.

42. Let $f : \mathbb{R} \to \mathbb{R}$ be a polynomial of the form $f(x) = a_0 + a_1 x + a_2 x^2$ with $a_0, a_1, a_2 \in \mathbb{R}$ and $a_2 \ne 0$, if

$$E_1 = \int_{-1}^{1} f(x)\,dx - [f(-1) + f(1)], \qquad E_2 = \int_{-1}^{1} f(x)\,dx - \frac{1}{2}(f(-1) + 2f(0) + f(1))$$

and $|x|$ is the absolute value of $x \in \mathbb{R}$, then —

(a) $|E_1| < |E_2|$ (b) $|E_1| = 2|E_2|$ (c) $|E_1| = 4|E_2|$ (d) $|E_1| = 8|E_2|$

43. The integral equation, $y(x) = \lambda \int_0^1 (3x-2)t\, y(t)\, dt$, with λ as a parameter, has

(a) only one characteristic number. (b) two characteristic number.
(c) more than two characteristic numbers. (d) no characteristic number.

44. Let $y(x)$ be a continuous solution of the initial value problem

$$y' + 2y = f(x),\ y(0) = 0, \text{ where } f(x) = \begin{cases} 1 & ; \quad 0 \le x \le 1 \\ 0 & ; \quad x > 1 \end{cases}$$

Then $y\left(\frac{3}{2}\right)$ is equal to :

(a) $\frac{\sinh(1)}{e^3}$ (b) $\frac{\cosh(1)}{e^3}$ (c) $\frac{\sinh(1)}{e^2}$ (d) $\frac{\cosh(1)}{e^2}$

45. Consider two weightless, inextensible rods *AB* and *BC*, suspended at *A* and joined by a flexible joint at *B*. Then the degrees of freedom of the system is :

(a) 3 (b) 4 (c) 5 (d) 6

46. The singular integral of the ODE $(xy' - y)^2 = x^2(x^2 - y^2)$ is :

(a) $y = x\sin x$ (b) $y = x\sin\left(x + \frac{\pi}{4}\right)$ (c) $y = x$ (d) $y = x + \frac{\pi}{4}$

47. Consider the initial value problem

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(0, y) = 4e^{-2y}$$

Then the value of $u(1,1)$ is :

(a) $4e^{-2}$ (b) $4e^2$ (c) $2e^{-4}$ (d) $4e^4$

48. The initial value problem $y' = 2\sqrt{y},\ y(0) = a$, has

(a) a unique solution if $a < 0$ (b) no solution if $a > 0$

(c) infinitely many solutions if $a = 0$ (d) a unique solution if $a \ge 0$

UNIT-4

49. Ten balls are put in 6 slots at random. Then the expected total number of balls in the two extreme slots is :

(a) $\frac{10}{6}$ (b) $\frac{10}{3}$ (c) $\frac{1}{6}$ (d) $\frac{6}{10}$

50. Let X, Y be independent random variables and let $Z = \frac{X-Y}{2} + 3$. If X has characteristic function ψ, then Z has characteristic function θ where

(a) $\theta(t) = e^{-i3t}\,\phi(2t)\,\psi(-2t)$ (b) $\theta(t) = e^{i3t}\,\phi\left(\frac{t}{2}\right)\psi\left(-\frac{t}{2}\right)$

(c) $\theta(t) = e^{-i3t}\,\phi\left(\frac{t}{2}\right)\psi\left(\frac{t}{2}\right)$ (d) $\theta(t) = e^{-i3t}\,\phi\left(\frac{t}{2}\right)\psi\left(-\frac{t}{2}\right)$

51. Suppose X_n, X are random variables such that X_n converges in distribution to X and $(-1)^n X_n$ also converges in distribution to X. Then,

(a) X must have a symmetric distribution. (b) X must be 0.

(c) X must have a density. (d) X^2 must be constant.

52. $\{N(t): t \geq 0\}$ is a Poisson process with rate $\lambda > 0$. Let $X_n = N(n),\ n = 0, 1, 2, \ldots$ Which of the following is ***correct*** ?

(a) $\{X_n\}$ is a transient Markov chain.

(b) $\{X_n\}$ is a recurrent Markov chain, but has no stationary distribution.

(c) $\{X_n\}$ has a stationary distribution.

(d) $\{X_n\}$ is an irreducible Markov chain.

53. Assume that $X \sim$ Binomial (n, p) for some $n \geq 1$ and $0 < p < 1$ and $Y \sim$ Poisson (λ) for some $\lambda > 0$. Suppose $E[X] = E[Y]$. Then

(a) $Var(X) = Var(Y)$

(b) $Var(X) < Var(Y)$

(c) $Var(Y) < Var(X)$

(d) $Var(X)$ may be larger or smaller than $Var(Y)$ depending on the values of n, p and λ.

54. Let $X_1, X_2, \ldots, X_7$ be a random sample from $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. Consider the problem of testing $H_0: \mu = 2$ against $H_1: \mu > 2$. Suppose the observed values of $x_1, x_2, \ldots, x_7$ are 1.2, 1.3, 1.7, 1.8, 2.1, 2.3, 2.7. If we use the Uniformly Most Powerful test, which of the following is true ?

(a) H_0 is accepted both at 5% and 1% levels of significance.

(b) H_0 is rejected both at 5% and 1% levels of significance.

(c) H_0 is rejected at 5% level of significance, but accepted at 1% level of significance.

(d) H_0 is rejected at 1% level of significance, but accepted at 5% level of significance.

55. Suppose $X_i \mid \theta_i \sim N(\theta_i, \sigma^2),\ i = 1, 2$ are independently distributed. Under the prior distribution, θ_1 and θ_2 are i.i.d. $N(\mu, \tau^2)$, where σ^2, μ and τ^2 are known. Then which of the following is true about the marginal distribution of X_1 and X_2 ?

(a) X_1 and X_2 are i.i.d. $N(\mu, \tau^2 + \sigma^2)$.

(b) X_1 and X_2 are not normally distributed.

(c) X_1 and X_2 are $N(\mu, \tau^2 + \sigma^2)$ but they are not independent.

(d) X_1 and X_2 are normally distributed but are not identically distributed.

56. Consider the model $Y_i = i\beta + \varepsilon_i, i = 1, 2, 3$ where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are independent with mean 0 and variance $\sigma^2, 2\sigma^2, 3\sigma^2$ respectively. Which of the following is the best linear unbiased estimate of β ?

(a) $\frac{y_1 + 2y_2 + 3y_3}{6}$ (b) $\frac{6}{11}\left(y_1 + \frac{y_2}{2} + \frac{y_3}{3}\right)$

(c) $\frac{y_1 + y_2 + y_3}{6}$ (d) $\frac{3y_1 + 2y_2 + y_3}{10}$

57. Let $Y = (Y_1, ..., Y_n)'$ have the multivariate normal distribution $N_n(0, I)$. Which of the following is the covariance matrix of the conditional distribution of Y given $\sum_{i=1}^{n} Y_i$?

[1 denotes the $n \times 1$ vector with all elements 1]

(a) I (b) $I + \frac{11'}{n}$ (c) $I - \frac{11'}{n}$ (d) $\frac{11'}{n}$

58. Suppose there are k strata of $N = kM$ units each with size M. Draw a sample of size n_i with replacement from the i-th stratum and denote by $\bar{y}_i$ the sample mean of the study variable selected in the i-th stratum, $i = 1, 2, ..., k$. Define $\bar{y}_s = \frac{1}{k}\sum_{i=1}^{k} \bar{y}_i$ and $\bar{y}_w = \frac{\sum_{i=1}^{k} n_i \bar{y}_i}{n}$. Which of the following is necessarily true ?

(a) $\bar{y}_s$ is unbiased but $\bar{y}_w$ is not unbiased for the population mean.

(b) $\bar{y}_s$ is not unbiased but $\bar{y}_w$ is unbiased for the population mean.

(c) Both $\bar{y}_s$ and $\bar{y}_w$ are unbiased for the population mean.

(d) Neither $\bar{y}_s$ nor $\bar{y}_w$ is unbiased for the population mean.

59. Consider a Balanced Incomplete Block Design (BIBD) with parameters (b, k, v, r, λ). Which of the following cannot possibly by the parameters of a BIBD ?

(a) $(b-1, k-\lambda, b-k, k, \lambda)$ (b) $(b, v-k, v, b-r, b-2r+\lambda)$

(c) $\left(\frac{v(v-1)}{2}, 2, v, v-1, 1\right)$ (d) $(k, b, r, v, \lambda - 1)$

60. Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables having an exponential distribution with mean $1/\lambda$. Let $S_n = X_1 + X_2 + \cdots + X_n$ and $N = \inf\{n \geq 1 : S_n > 1\}$. Then $Var(N)$ equals

(a) 1 (b) λ (c) λ^2 (d) ∞

PART - C (Mathematical Sciences)

UNIT-1

61. For $n \geq 1$, let $g_n(x) = \sin^2\left(x + \frac{1}{n}\right)$, $x \in [0, \infty)$ and $f_n(x) = \int_0^x g_n(t)\,dt$. Then,

(a) $\{f_n\}$ converges pointwise to a function f on $[0, \infty)$, but does not converge uniformly on $[0, \infty)$.

(b) $\{f_n\}$ does not converge pointwise to any function on $[0, \infty)$.

(c) $\{f_n\}$ converges uniformly on [0, 1].

(d) $\{f_n\}$ converges uniformly on $[0, \infty)$.

62. Let 'a' be a positive real number. Which of the following integrals are convergent ?

(a) $\int_0^a \frac{1}{x^4}\,dx$ (b) $\int_0^a \frac{1}{\sqrt{x}}\,dx$ (c) $\int_4^\infty \frac{1}{x \log_e x}\,dx$ (d) $\int_5^\infty \frac{1}{x(\log_e x)^2}\,dx$

63. Which of the following sets in $\mathbb{R}^2$ have positive Lebesgue measure ?

$\left[\text{For two sets } A, B \subseteq \mathbb{R}^2, A + B = \{a + b \mid a \in A, b \in B\}\right]$

(a) $S = \{(x, y) \mid x^2 + y^2 = 1\}$

(b) $S = \{(x, y) \mid x^2 + y^2 < 1\}$

(c) $S = \{(x, y) \mid x = y\} + \{(x, y) \mid x = -y\}$

(d) $S = \{(x, y) \mid x = y\} + \{(x, y) \mid x = y\}$

64. Which of the following sets of functions are uncountable ? [$\mathbb{N}$ stands for the set of natural numbers]

(a) $\{f \mid f : \mathbb{N} \to \{1, 2\}\}$

(b) $\{f \mid f : \{1, 2\} \to \mathbb{N}\}$

(c) $\{f \mid f : \{1, 2\} \to \mathbb{N},\ f(1) \leq f(2)\}$

(d) $\{f \mid f : \mathbb{N} \to \{1, 2\},\ f(1) \leq f(2)\}$

65. Let f be a bounded functions on $\mathbb{R}$ and $a \in \mathbb{R}$. For $\delta > 0$, let $\omega(a, \delta) = \sup |f(x) - f(a)|$, $x \in [a - \delta, a + \delta]$. Then,

(a) $\omega(a, \delta_1) \leq \omega(a, \delta_2)$ if $\delta_1 \leq \delta_2$

(b) $\lim_{\delta \to 0^+} \omega(a, \delta) = 0$ for all $a \in \mathbb{R}$

(c) $\lim_{\delta \to 0^+} \omega(a, \delta)$ need not exist

(d) $\lim_{\delta \to 0^+} \omega(a, \delta) = 0$ if and only if f is continuous at a

66. For $n \geq 2$, let $a_n = \frac{1}{n \log n}$. Then,

(a) The sequence $\{a_n\}_{n=2}^\infty$ is convergent.

(b) The series $\sum_{n=2}^\infty a_n$ is convergent.

(c) The series $\sum_{n=2}^\infty a_n^2$ is convergent.

(d) The series $\sum_{n=2}^\infty (-1)^n a_n$ is convergent.

67. Let $\{a_0, a_1, a_2, ...\}$ be a sequence of real numbers. For any $k \geq 1$, let $s_n = \sum_{k=0}^{n} a_{2k}$. Which of the following statements are correct ?

(a) If $\lim_{n\to\infty} s_n$ exists, then $\sum_{m=0}^{\infty} a_m$ exists.

(b) If $\lim_{n\to\infty} s_n$ exists, then $\sum_{m=0}^{\infty} a_m$ need not exist.

(c) If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} s_n$ exists.

(d) If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} s_n$ need not exist.

68. Let $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be the function $F(x, y) = \langle Ax, y\rangle$, where $\langle , \rangle$ is the standard inner product of $\mathbb{R}^n$ and A is a $n \times n$ real matrix. Here, D denotes the total derivative. Which of the following statements are correct?

(a) $(DF(x, y))(u, v) = \langle Au, y\rangle + \langle Ax, v\rangle$

(b) $(DF(x, y))(0, 0) = 0$

(c) $DF(x, y)$ may not exist for some $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$

(d) $DF(x, y)$ does not exist at $(x, y) = (0, 0)$

69. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous function such that $\int_{\mathbb{R}^n} |f(x)| dx < \infty$. Let A be a real $n \times n$ invertible matrix and for $x, y \in \mathbb{R}^n$, let $\langle x, y\rangle$ denote the standard inner product in $\mathbb{R}^n$. Then $\int_{\mathbb{R}^n} f(Ax) e^{i\langle y, x\rangle} dx =$

(a) $\int_{\mathbb{R}^n} f(x) e^{i\langle (A^{-1})^T y, x\rangle} \frac{dx}{|\det A|}$

(b) $\int_{\mathbb{R}^n} f(x) e^{i\langle A^T y, x\rangle} \frac{dx}{|\det A|}$

(c) $\int_{\mathbb{R}^n} f(x) e^{i\langle (A^T)^{-1} y, x\rangle} dx$

(d) $\int_{\mathbb{R}^n} f(x) e^{i\langle A^{-1} y, x\rangle} \frac{dx}{|\det A|}$

70. An $n \times n$ complex matrix A satisfies $A^k = I_n$, the $n \times n$ identity matrix, where k is a positive integer >1. Suppose 1 is not an eigenvalue of A. Then which of the following statements are necessarily true ?

(a) A is diagonalizable

(b) $A + A^2 + \cdots + A^{k-1} = 0$, the $n \times n$ zero matrix.

(c) $tr(A) + tr(A^2) + \cdots + tr(A^{k-1}) = -n$

(d) $A^{-1} + A^{-2} + \cdots + A^{-(k-1)} = -I_n$

71. Let S be the set of 3×3 real matrices A with $A^T A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Then the set S contains

(a) a nilpotent matrix.

(b) a matrix of rank one.

(c) a matrix of rank two.

(d) a non-zero skew-symmetric matrix.

72. Let $S : \mathbb{R}^n \to \mathbb{R}^n$ be given by $S(v) = \alpha v$ for a fixed $\alpha \in \mathbb{R}$, $\alpha \neq 0$. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation such that $B = \{v_1, ..., v_n\}$ is a set of linearly independent eigenvectors of T. Then,

(a) The matrix of T with respect to B is diagonal.

(b) The matrix of $(T - S)$ with respect to B is diagonal.

(c) The matrix of T with respect to B is not necessarily diagonal, but is upper triangular.

(d) The matrix of T with respect to B is diagonal but the matrix of $(T - S)$ with respect to B is not diagonal.

73. Let $A = \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix}$ be a 3×3 matrix where a, b, c, d are integers. Then, we must have:

(a) If $a \neq 0$, there is a polynomial $p \in \mathbb{Q}[x]$ such that $p(A)$ is the inverse of A.

(b) For each polynomial $q \in \mathbb{Z}[x]$, the matrix $q(A) = \begin{pmatrix} q(a) & q(b) & q(c) \\ 0 & q(a) & q(d) \\ 0 & 0 & q(a) \end{pmatrix}$

(c) If $A^n = 0$ for some positive integer n, then $A^3 = 0$.

(d) A commutes with every matrix of the form $\begin{pmatrix} a' & 0 & c' \\ 0 & a' & 0 \\ 0 & 0 & a' \end{pmatrix}$.

74. Let $p_n(x) = x^n$ for $x \in \mathbb{R}$ and let $\wp = span\{p_0, p_1, p_2, ...\}$. Then,

(a) $\wp$ is the vector space of all real valued continuous functions on $\mathbb{R}$.

(b) $\wp$ is a subspace of all real valued continuous functions on $\mathbb{R}$.

(c) $\{p_0, p_1, p_2, ...\}$ is a linearly independent set in the vector space of all continuous functions on $\mathbb{R}$.

(d) Trigonometric functions belong to $\wp$.

75. Which of the following are subspaces of the vector space $\mathbb{R}^3$?

(a) $\{(x, y, z) : x + y = 0\}$

(b) $\{(x, y, z) : x - y = 0\}$

(c) $\{(x, y, z) : x + y = 1\}$

(d) $\{(x, y, z) : x - y = 1\}$

76. Let A be an invertible 4×4 real matrix. Which of the following are NOT true ?

(a) Rank $A = 4$

(b) For every vector $b \in \mathbb{R}^4$, $Ax = b$ has exactly one solution.

(c) dim (null space A) ≥ 1

(d) 0 is an eigenvalue of A.

77. Consider non-zero vector spaces V_1, V_2, V_3, V_4 and linear transformations $\phi_1 : V_1 \to V_2$, $\phi_2 : V_2 \to V_3$, $\phi_3 : V_3 \to V_4$ such that $\ker(\phi_1) = \{0\}$, Range $(\phi_1) = \ker(\phi_2)$. Range $(\phi_2) = \ker(\phi_3)$, Range $(\phi_3) = V_4$. Then

(a) $\sum_{i=1}^{4}(-1)^i \dim V_i = 0$ (b) $\sum_{i=2}^{4}(-1)^i \dim V_i > 0$

(c) $\sum_{i=1}^{4}(-1)^i \dim V_i < 0$ (d) $\sum_{i=1}^{4}(-1)^i \dim V_i \neq 0$

78. Let $\underline{u}$ be a real $n \times 1$ vector satisfying $\underline{u}'\underline{u} = 1$, where $\underline{u}'$ is the transpose of $\underline{u}$. Define $A = I - 2\underline{u}\,\underline{u}'$, where I is the n-th order identity matrix. Which of the following statements are true ?

(a) A is singular (b) $A^2 = A$ (c) $\text{Trace}(A) = n-2$ (d) $A^2 = I$

UNIT-2

79. Let f be an entire function. Which of the following statements are correct ?

(a) f is constant if the range of f is contained in a straight line.

(b) f is constant if f has uncountably many zeros.

(c) f is constant if f is bounded on $\{z \in \mathbb{C} : \text{Re}(z) \le 0\}$.

(d) f is constant if the real part of f is bounded.

80. Consider the following subsets of the complex plane:

$$\Omega_1 = \left\{ C \in \mathbb{C} : \begin{bmatrix} 1 & C \\ \bar{C} & 1 \end{bmatrix} \text{is non-negative definite (or equivalently positive semi-definite)} \right\}$$

$$\Omega_2 = \left\{ C \in \mathbb{C} : \begin{bmatrix} 1 & C & C \\ \bar{C} & 1 & C \\ \bar{C} & \bar{C} & 1 \end{bmatrix} \text{is non-negative definite (or equivalently positive semi-definite)} \right\}$$

Let $\bar{D} = \{z \in \mathbb{C} \,|\, |z| \le 1\}$. Then,

(a) $\Omega_1 = \bar{D}, \Omega_2 = \bar{D}$ (b) $\Omega_1 \neq \bar{D}, \Omega_2 = \bar{D}$ (c) $\Omega_1 = \bar{D}, \Omega_2 \neq \bar{D}$ (d) $\Omega_1 \neq \bar{D}, \Omega_2 \neq \bar{D}$

81. Let p be a polynomial in 1-complex variable. Suppose all zeros of p are in the upper half plane $H = \{ z \in \mathbb{C} \,|\, \text{Im}(z) > 0\}$. Then,

(a) $\text{Im}\dfrac{p'(z)}{p(z)} > 0$ for $z \in \mathbb{R}$ (b) $\text{Re}\, i\dfrac{p'(z)}{p(z)} < 0$ for $z \in \mathbb{R}$

(c) $\text{Im}\dfrac{p'(z)}{p(z)} > 0$ for $z \in \mathbb{C}$, with $\text{Im}\, z < 0$ (d) $\text{Im}\dfrac{p'(z)}{p(z)} > 0$ for $z \in \mathbb{C}$, with $\text{Im}\, z > 0$

82. Let f be an analytic function defined on the open unit disc in $\mathbb{C}$. Then f is constant if—

(a) $f\left(\dfrac{1}{n}\right) = 0$ for all $n \ge 1$ (b) $f(z) = 0$ for all $|z| = \dfrac{1}{2}$

(c) $f\left(\dfrac{1}{n^2}\right) = 0$ for all $n \ge 1$ (d) $f(z) = 0$ for all $z \in (-1, 1)$

83. Let $\sigma:\{1,2,3,4,5\}\to\{1,2,3,4,5\}$ be a permutation (one-to-one and onto function) such that $\sigma^{-1}(j)\le\sigma(j)$ $\forall\ j, 1\le j\le 5$. Then which of the following are true ?

(a) $\sigma\circ\sigma(j)=j$ for all $j, 1\le j\le 5$

(b) $\sigma^{-1}(j)=\sigma(j)$ for all $j, 1\le j\le 5$

(c) The set $\{k:\sigma(k)\ne k\}$ has an even number of elements.

(d) The set $\{k:\sigma(k)=k\}$ has an odd number of elements.

84. Which of the following primes satisfy the congruence $a^{24}\equiv 6a+2 \bmod 13$?

(a) 41 (b) 47 (c) 67 (d) 83

85. If x, y and z are elements of a group such that $xyz=1$, then

(a) $yzx=1$ (b) $yxz=1$ (c) $zxy=1$ (d) $zyx=1$

86. Which of the following cannot be the class equation of a group of order 10 ?

(a) 1 + 1 + 1 + 2 + 5 = 10
(b) 1 + 2 + 3 + 4 = 10
(c) 1 + 2 + 2 + 5 = 10
(d) 1 + 1 + 2 + 2 + 2 + 2 = 10

87. Let $C([0,1])$ be the ring of all real valued continuous functions on [0, 1]. Which of the following statements are true ?

(a) $C([0,1])$ is an integral domain.

(b) The set of all functions vanishing at 0 is a maximal ideal.

(c) The set of all functions vanishing at both 0 and 1 is a prime ideal.

(d) If $f\in C([0,1])$ is such that $(f(x))^n=0$ for all $x\in[0,1]$ for some $n>1$, then $f(x)=0$ for all $x\in[0,1]$.

88. Which of the following polynomials are irreducible in the ring $\mathbb{Z}[x]$ of polynomials in one variable with integer coefficients ?

(a) x^2-5
(b) $1+(x+1)+(x+1)^2+(x+1)^3+(x+1)^4$
(c) $1+x+x^2+x^3+x^4$
(d) $1+x+x^2+x^4$

89. Determine which of the following polynomials are irreducible over the indicated rings.

(a) $x^5-3x^4+2x^3-5x+8$ over $\mathbb{R}$
(b) x^3+2x^2+x+1 over $\mathbb{Q}$
(c) x^3+3x^2-6x+3 over $\mathbb{Z}$
(d) x^4+x^2+1 over $\mathbb{Z}/2\mathbb{Z}$

90. Consider the set $\mathbb{Z}$ of integers, with the topology τ in which a subset is closed if and only if it is empty, or $\mathbb{Z}$, or finite. Which of the following statements are true ?

(a) τ is the subspace topology induced from the usual topology on $\mathbb{R}$.

(b) $\mathbb{Z}$ is compact in the topology τ.

(c) $\mathbb{Z}$ is Hausdorff in the topology τ.

(d) Every infinite subset of $\mathbb{Z}$ is dense in the topology τ.

UNIT-3

91. For the initial value problem

$$\frac{dy}{dx} = y^2 + \cos^2 x, \quad x > 0$$

$$y(0) = 0$$

The largest interval of existence of the solution predicted by Picard's theorem is :

(a) [0, 1] (b) [0, 1/2] (c) [0, 1/3] (d) [0, 1/4]

92. For an arbitrary continuously differentiable function f, which of the following is a general solution of $z(px - qy) = y^2 - x^2$

(a) $x^2 + y^2 + z^2 = f(xy)$ (b) $(x+y)^2 + z^2 = f(xy)$

(c) $x^2 + y^2 + z^2 = f(y-x)$ (d) $x^2 + y^2 + z^2 = f((x+y)^2 + z^2)$

93. Let P be a continuous function on $\mathbb{R}$ and W the Wronskian of two linearly independent solutions y_1 and y_2 of the ODE.

$$\frac{d^2y}{dx^2} + (1+x^2)\frac{dy}{dx} + P(x)y = 0, \; x \in \mathbb{R}$$

Let $W(1) = a, W(2) = b$ and $W(3) = c$, then

(a) $a < 0$ and $b > 0$ (b) $a < b < c$ or $a > b > c$

(c) $\frac{a}{|a|} = \frac{b}{|b|} = \frac{c}{|c|}$ (d) $0 < a < b$ and $b > c > 0$

94. The following numerical integration formula is exact for all polynomials of degree less than or equal to 3.

(a) Trapezoidal rule (b) Simpson's 1/3rd rule

(c) Simpson's 3/8th rule (d) Gauss-Legendre 2 point formula

95. A particle of mass m is contrained to move on the surface of a cylinder $x^2 + y^2 = a^2$ under the influence of a force directed towards the origin and proportional to the distance of the particle from the origin. Then,

(a) the angular momentum about z-axis is constant.

(b) the angular momentum about z-axis is not constant.

(c) the motion is simple harmonic in z-direction.

(d) the motion is not simple harmonic in z-direction.

96. The critical point of the system $\frac{dx}{dt} = -4x - y, \; \frac{dy}{dt} = x - 2y$ is an

(a) asymptotically stable node (b) unstable node

(c) asymptotically stable spiral (d) unstable spiral

97. The function $G(x,\zeta)=\begin{cases} a+b\log\zeta, & 0<x\le\zeta \\ c+d\log x, & \zeta\le x\le 1\end{cases}$

is a Green's function for $xy''+y'=0$, subject in y being bounded as $x\to 0$ and $y(1)=y'(1)$, if

(a) $a=1,\ b=1,\ c=1,\ d=1$
(b) $a=1,\ b=0,\ c=1,\ d=0$
(c) $a=0,\ b=1,\ c=0,\ d=1$
(d) $a=0,\ b=0,\ c=0,\ d=0$

98. For the integral equation, $y(x)=1+x^3+\int_0^x K(x,t)\,y(t)\,dt$ with kernel $K(x,t)=2^{x-t}$, the iterated kernel $K_3(x,t)$ is:

(a) $2^{x-t}(x-t)^2$
(b) $2^{x-t}(x-t)^3$
(c) $2^{x-t-1}(x-t)^2$
(d) $2^{x-t-1}(x-t)^3$

99. The extremal of the functional $I=\int_0^{x_1} y^2(y')^2\,dx$ that passes through (0, 0) and (x_1, y_1) is :

(a) a constant function
(b) a linear function of x
(c) part of a parabola
(d) part of an ellipse

100. The extremal of the functional $\int_0^{\alpha}(y'^2-y^2)\,dx$ that passes through (0, 0) and $(\alpha, 0)$ has a

(a) weak minimum if $\alpha<\pi$
(b) strong minimum if $\alpha<\pi$
(c) weak minimum if $\alpha>\pi$
(d) strong minimum if $\alpha>\pi$

101. The second order partial differential equation, $u_{xx}+xu_{yy}=0$ is

(a) elliptic for $x>0$
(b) hyperbola for $x>0$
(c) elliptic for $x<0$
(d) hyperbolic for $x<0$

102. Which of the following are complete integrals of the partial differential equations, $pqx+yq^2=1$?

(a) $z=\dfrac{x}{a}+\dfrac{ay}{x}+b$
(b) $z=\dfrac{x}{b}+\dfrac{ay}{x}+b$
(c) $z^2=4(ax+y)+b$
(d) $(z-b)^2=4(ax+y)$

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(Answer Key) — PART - B (Mathematical Sciences)

21. (a)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (b)
26. (c)	**27.** (d)	**28.** (d)	**29.** (b)	**30.** (b)
31. (c)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (b)
36. (c)	**37.** (c)	**38.** (b)	**39.** (a)	**40.** (b)
41. (d)	**42.** (a)	**43.** (d)	**44.** (b)	**45.** (b)
46. (b)	**47.** (c)	**48.** (b)	**49.** (d)	**50.** (d)
51. (c)	**52.** (b)	**53.** (d)	**54.** (d)	**55.** (b)
56. (b)	**57.** (b)	**58.** (a)	**59.** (c)	**60.** (b)

(Answer Key) — PART - C (Mathematical Sciences)

61. (a), (c)	**62.** (a), (c)	**63.** (c), (d)	**64.** (a), (b), (c), (d)	**65.** (a), (b)
66. (d)	**67.** (c), (d)	**68.** (a), (b), (c)	**69.** (*)	**70.** (a), (c)
71. (c), (d)	**72.** (a), (b), (c), (d)	**73.** (a), (b)	**74.** (a), (b), (c), (d)	**75.** (a), (c)
76. (b), (d)	**77.** (a), (c)	**78.** (a), (c), (d)	**79.** (a), (d)	**80.** (b), (d)
81. (a)	**82.** (a), (b), (d)	**83.** (c), (d)	**84.** (c), (d)	**85.** (c), (d)
86. (d)	**87.** (b), (c)	**88.** (a), (b), (c), (d)	**89.** (a), (d)	
90. (a), (b), (c), (d)				
91. (a), (b)	**92.** (b), (d)	**93.** (a), (b), (c), (d)	**94.** (a), (b), (c)	**95.** (c)
96. (c)	**97.** (a), (b), (c), (d)	**98.** (d)	**99.** (b), (c)	**100.** (a),(c)
101. (a), (c)	**102.** (c), (d)			