CSIR-NET-(P YQ) MATHEMATICAL SCIENCE
JUNE-2014-I

SECTION-B

21. Let A be a 5×5 matrix with real entries such that the sum of the entries in each row of A is 1. Then the sum of all the entries in A^3 is

(a) 3 (b) 15 (c) 5 (d) 125

Ans. **(c)**

22. Given the permutation $\sigma=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 3 & 1 & 2 & 5 & 4\end{pmatrix}$, the matrix A is defined to be the one whose *i*-th column is the $\sigma(i)$-th column of the identity matrix I. Which of the following is correct ?

(a) $A=A^{-2}$ (b) $A=A^{-4}$ (c) $A=A^{-5}$ (d) $A=A^{-1}$

Ans. **(c)**

23. Let J denote a 101×101 matrix with all the entries equal to 1 and let I denote the identity matrix of order 101. then the determinant of $J-I$ is

(a) 101 (b) 1 (c) 0 (d) 100

Ans. **(d)**

24. Let $A\subseteq\mathbb{R}$ and $f:A\to\mathbb{R}$ be given by $f(x)=x^2$. then f is uniformly continuous if

(a) A is a bounded subset of $\mathbb{R}$

(b) A is a dense subset of $\mathbb{R}$

(c) A is an unbounded and connected subset of $\mathbb{R}$

(d) A is an unbounded and open subset of $\mathbb{R}$

Ans. **(a)**

25. Let α,p be real numbers and $\alpha > 1$

(a) If $p>1$ then $\int_{-\infty}^{\infty}\frac{1}{|x|^{p\alpha}}dx<\infty$

(b) If $p>\frac{1}{\alpha}$ then $\int_{-\infty}^{\infty}\frac{1}{|x|^{p^{\alpha}}}dx<\infty$

(c) If $p<\frac{1}{\alpha}$ then $\int_{-\infty}^{\infty}\frac{1}{|x|^{p\alpha}}dx<\infty$

(d) For any $p\in\mathbb{R}$ we have $\int_{-\infty}^{\infty}\frac{1}{|x|^{p\alpha}}dx=\infty$

Ans. **(d)**

26. Let $f:X\to Y$ be function from a metric space X to another metric space Y. For any Cauchy sequence $\{x_n\}$ in X,

(a) If f is continuous then $\{f(x_n)\}$ is a Cauchy sequence in Y.

(b) If $\{f(x_n)\}$ is Cauchy then $\{f(x_n)\}$ is always convergent in Y

(c) If $\{f(x_n)\}$ is Cauchy in Y then f is continuous

(d) $\{x_n\}$ is always convergent in X.

Ans. **(c)**

27. Le $M_{m\times n}(\mathbb{R})$ be the set of all m×n matrices with real entries. Which of the following statements is correct ?

(a) There exists $A \in M_{2\times 5}(\mathbb{R})$ such that the dimension of the null space of A is 2

(b) There exists $A \in M_{2\times 5}(\mathbb{R})$ such that the dimension of the null space of A is 0

(c) There exists $A \in M_{2\times 5}(\mathbb{R})$ and $B \in M_{5\times 2}(\mathbb{R})$ such that AB is the 2×2 identity matrix

(d) There exists $A \in M_{2\times 5}(\mathbb{R})$ whose null space is $\{(x_1, x_2, x_3, x_5) \in \mathbb{R}^5 : x_1 = x_2, x_3 = x_4 = x_5\}$

Ans. **(c)**

28. $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + ... + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}\right)$ equals

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}+1$ (d) $\frac{1}{\sqrt{2}+1}$

Ans. **(b)**

29. Consider the following sets of functions on $\mathbb{R}$

W = The set of constant functions on $\mathbb{R}$

X = The set of polynomial functions on $\mathbb{R}$

Y = The set of continuous functions on $\mathbb{R}$

Z = The set of all functions on R. Which of these sets has the same cardinality as that of $\mathbb{R}$

(a) Only *W*.
(b) Only *W* and *X*
(c) Only *W*, *X* and *Z*
(d) All of *W*, *X*, *Y* and *Z*

Ans. **(*)**

30. Let $p(x)$ be a polynomial in the real variable x of degree 5. Then $\lim\limits_{n\to\infty} \frac{p(n)}{2^n}$ is

(a) 5 (b) 1 (c) 0 (d) ∞

Ans. **(c)**

31. For a continuous function $f : \mathbb{R} \to \mathbb{R}$, let $Z(f) = \{x \in \mathbb{R} : f(x) = 0\}$. Then $Z(f)$ is always

(a) compact (b) open (c) connected (d) closed

Ans. **(d)**

32. For the matrix A as given below, which of them satisfy $A^6 = I$

(a) $A = \begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0 \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ 0 & -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$

(c) $A = \begin{pmatrix} \cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{6} & 0 & \cos\frac{\pi}{6} \end{pmatrix}$

(d) $A = \begin{pmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0 \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Ans. **(b)**

33. If n is a positive integer such that the sum of all positive integers a satisfying $1 \le a \le n$ and GCD (a,n) =1 is equal to 240 n, then the number of summands, namely, $\varphi(n)$, is

(a) 120 (b) 124 (c) 240 (d) 480

Ans. **(d)**

34. The total number of non-isomorphic groups of order 122 is

(a) 2 (b) 1 (c) 61 (d) 4

Ans. **(a)**

35. An ice cream shop sells ice creams in five possible flavours: Vanilla, Chocolate, Strawberry, Mango and Pineapple How many combinations of three scoop cones are possible ?

[Note: the repetition of flavours is allowed but the order in which the flavours are chosen does not matter]

(a) 10 (b) 20 (c) 35 (d) 243

Ans. **(c)**

36. Let $A \subseteq \mathbb{R}^2$ and $X = \mathbb{R}^2 \setminus A$ be subsets with subspace topology inherited from the usual topology on $\mathbb{R}^2$. Then

(a) A is countable dense implies that X is totally disconnected.

(b) A is unbounded implies that X is compact.

(c) A is open implies that X is compact.

(d) A is countable implies that X is path connected

Ans. **(d)**

37. Let f,g be meromorphic functions on $\mathbb{C}$. If f has a zero of order k at $z = a$ and g has a pole of order m at $z = 0$, then $g(f(z))$ has

(a) a zero of order km at $z = a$ (b) a pole of order km at $z = a$

(c) a zero of order $|k - m|$ at $z = a$ (d) a pole of order $|k - m|$ at $z = a$

Ans. **(b)**

38. Let $p(x)$ be a polynomial of the real variable x of degree $k \ge 1$. Consider the power series

$f(z) = \sum_{n=0}^{\infty} p(n) z^n$ where z is a complex variable. Then the radius of convergence of $f(z)$ is

(a) 0 (b) 1 (c) k (d) ∞

Ans. **(b)**

39. Let G denote the group of all the automorphisms of the field $F_{3^{100}}$ that consists of 3^{100} elements. Then the number of distinct subgroups of G is equal to

(a) 4 (b) 3 (c) 100 (d) 9

Ans. **(d)**

40. Let p,q be distinct primes, Then

(a) $\mathbb{Z}/p^2q\mathbb{Z}$ has exactly 3 distinct ideals (b) $\mathbb{Z}/p^2q\mathbb{Z}$ has exactly 3 distinct prime ideals

(c) $\mathbb{Z}/p^2q\mathbb{Z}$ has exactly 2 distinct prime ideals (d) $\mathbb{Z}/p^2q\mathbb{Z}$ has unique maximal ideals

Ans. **(c)**

41. The homogeneous integral equation

$\varphi(x) - \lambda\int_0^1 (3x-2)t \ \varphi(t)\,dt = 0$ has

(a) one characteristic number (b) three characteristic numbers

(c) two characteristic numbers (d) No characteristic number

Ans. **(d)**

42. The initial value problem

$$\frac{\partial u}{\partial t}+x\frac{\partial u}{\partial x}=x, 0\le x\le 1,\ t>0 \text{ and } u(x,0)=2x \text{ has}$$

(a) a unique solution $u(x, t)$ which $\to\infty$ as $t\to\infty$

(b) more than one solution

(c) a solution which remains bounded as $t\to\infty$

(d) no solution

Ans. **(c)**

43. Let $Y_1(x)$ and $Y_2(x)$ defined on $[0,1]$ be twice continuously differentiable functions satisfying

$Y''(x)+Y'(x)+Y(x)=0$ Let $W(x)$ be the Wronskian of Y_1 and Y_2 and satisfy $W\left(\frac{1}{2}\right)=0$. Then

(a) $W(x)=0$ for $x\in[0,1]$

(b) $W(x)\neq 0$ for $x\in[0,1/2)\cup\left(\frac{1}{2},1\right]$

(c) $W(x)>0$ for $x\in(1/2,1]$

(d) $W(x)<0$ for $x\in[0,1/2)$

Ans. **(a)**

44. Consider two waves of same angular frequency ω, same angular wave number k, same amplitude a traveling in the positive direction of x– axis with the same speed and with phase difference ϕ, Then the superpositive principle yields a resultant wave with

(a) Amplitude $2a$ and phase ϕ

(b) Amplitude $2a$ and phase $(\phi/2)$

(c) Amplitude $2a\cos(\phi/2)$ and phase$(\phi/2)$

(d) Amplitude $2a\cos(\phi/2)$ and phase

Ans. **(c)**

45. Let $x=x(s),\ y=y(s), u=u(s), s\in\mathbb{R}$ be the characteristic curve of the PDE $\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)-u=0$ passing through a given curve

$$x=0, y=\tau, u=\tau^2, \tau\in\mathbb{R}$$

Then the characteristics are given by

(a) $x=3\tau\left(e^{s}-1\right), y=\frac{\tau}{2}\left(e^{-s}+1\right), u=\tau^2e^{-2s}$

(b) $x=2\tau\left(e^{-s}-1\right), y=\tau\left(2e^{2s}-1\right), u=\frac{\tau^2}{2}\left(1+e^{-2s}\right)$

(c) $x=2\tau\left(e^{s}-1\right), y=\frac{\tau}{2}\left(e^{s}+1\right), u=\tau^2e^{2s}$

(d) $x=\tau\left(e^{-s}-1\right), y=-2\tau\left(e^{-s}+\frac{3}{2}\right), u=\tau^2\left(2e^{-2s}-1\right)$

Ans. **(c)**

46. Let $f(x)=ax+b$ for $a,b\in\mathbb{R}$. Then the iteration $x_{n+1}=f(x_n)$ starting from any given x_0 for $n\geq 0$ converges

(a) for all $a\in\mathbb{R}$ (b) for no $a\in\mathbb{R}$ (c) for $a\in[0,1)$ (d) only for $a=0$

Ans. (c)

47. Consider the initial value problem in $\mathbb{R}^2$

$$Y'(t)=AY+BY;\quad Y(0)=Y_0,\ \text{where } A=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}, B=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$$

Then $Y(t)$ is given by

(a) $e^{tA}e^{tB}Y_0$ (b) $e^{tB}e^{tA}Y_0$ (c) $e^{t(A+B)}Y_0$ (d) $e^{-t(A+B)}Y_0$

Ans. (c)

48. The curve extremizing the functional

$$I(y)=\int_1^2\sqrt{\frac{1+(y'(x))^2}{x}}dx,$$

$y(1)=0,\ y(2)=1$ is

(a) an ellipse (b) a parabola (c) a circle (d) a straight line

Ans. (c)

SECTION-C

61. Let V denote a vector space over of Field F and with a basis $B=\{e_1,e_2,...e_n\}$. Let $x_1,x_2,...x_n\in F$. Let

$C=\{x_1e_1, x_1e_1+x_2e_2,...,x_1e_1+x_2e_2+....+x_ne_n\}$. Then

(a) C is a linearly independent set implies that $x_i\neq 0$ for every $i=1,2,....,n$.

(b) $x_i\neq 0$ for every $i=1,2,....,n$ implies that C is a linearly independent set

(c) The linear span C is V implies that $x_i\neq 0$ for every $i=1,2,....,n$.

(d) $x_i\neq 0$ for every $i=1,2,....,n$ implies that linear span of C is V

Ans. (a, b,c,d)

62. Let V denote the vector space of all polynomials over $\mathbb{R}$ of degree less than or equal to n. Which of the following defines a norm on V?

(a) $\|p\|^2=|p(1)|^2+....+|p(n+1)|^2,\ p\in V$ (b) $\|p\|=\sup_{t\in[0,1]}|p(t)|,\ p\in V$

(c) $\|p\|=\int_0^1|p(t)|dt,\ p\in V$ (d) $\|p\|=\sup_{t\in[0,1]}|p'(t)|,\ p\in V$

Ans. (a, b,c)

63. Let u,v,w be vectors in an inner product space V, satisfying $\|u\|=\|v\|=\|w\|=2$ and

$\langle u,v\rangle=0, \langle u,w\rangle=1, \langle v,w\rangle=-1$. Then which of the following are True ?

(a) $\|w+v-u\|=2\sqrt{2}$

(b) $\left\{\frac{1}{2}u,\frac{1}{2}v\right\}$ forms an orthonormal basis of a two dimensional subspace of V

(c) w and $4u - w$ are orthogonal to each other

(d) u, v, w are necessarily linearly independent

Ans. **(a, b,c,d)**

64. Let A be a 4×4 matrix over $\mathbb{C}$ such that rank (A) = 2 and $A^3 = A^2 \neq 0$.
Suppose that A is not diagonalizable. Then

(a) One of the Jordan blocks of the Jordan canonical from of A is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(b) $A^2 = A \neq 0$

(c) There exists a vector v such that $Av \neq 0$ but $A^2 v = 0$

(d) The characteristic polynomial of A is $x^4 - x^3$

Ans. **(a,c,d)**

65. Let $l^2 = \left\{ x = (x_1, x_2, ...) : x_n \in \mathbb{C} \ \ \forall n \geq 1 \text{ and } \Sigma_{n=1}^{\infty} |x_n|^2 < \infty \right\}$ and $e_n \in l^2$ be the sequence whose n-th element is 1 and all other elements are zero. Equip the space l^2 with the norm $\|x\| = \sqrt{\Sigma_{n=1}^{\infty} |x_n|^2}$. Then the set $S = \{e_n : n \geq 1\}$

(a) is closed

(b) is bounded

(c) is compact

(d) contains a convergent subsequence

Ans. **(a,b)**

66. Let $\varphi : \mathbb{R}^2 \to \mathbb{C}$ be the map $\varphi(x, y) = z$, where $z = x + iy$. Let $f : \mathbb{C} \to \mathbb{C}$ be the function $f(z) = z^2$ and $F = \varphi^{-1} f \varphi$. Which of the following are correct

(a) The linear transformation $T(x, y) = 2\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ represents the derivative of F at (x, y)

(b) The linear transformation $T(x, y) = 2\begin{pmatrix} x & y \\ y & x \end{pmatrix}$ represents the derivative of F at (x,y)

(c) The linear transformation $T(z) = 2z$ represents the derivative of f at $z \in \mathbb{C}$

(d) The linear transformation $T(z) = 2z$ represents the derivative of f only at 0

Ans. **(a,c)**

67. Let $X = \left\{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 < 5 \right\}$ and $K = \left\{ (x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 2 \text{ or } 3 \leq x^2 + y^2 \leq 4 \right\}$. Then,

(a) $X \setminus$ K has three connected components.

(b) $X \setminus$ K has no relatively compact connected component in X

(c) $X \setminus$ K has two relatively compact connected component in X

(d) All connected components of X\K are relatively compact in X

Ans. **(a,c)**

68. For two subset X and Y of $\mathbb{R}$, let $X + Y = \{x + y : x \in X, y \in Y\}$

(a) If X and Y are open sets then X+Y is open

(b) If X and Y are closed sets then X+Y is closed

(c) If X and Y are compact sets then X+Y is compact

(d) If X is closed and Y is compact then X+Y is closed

Ans. **(a,c,d)**

69. Let $\{f_n\}$ be a sequence of continuous functions on $\mathbb{R}$.

(a) If $\{f_n\}$ converges to f pointwise on $\mathbb{R}$, then $\lim\limits_{n\to\infty}\int_{-\infty}^{\infty} f_n(x)\,dx = \int_{-\infty}^{\infty} f(x)\,dx$

(b) If $\{f_n\}$ converges to f uniformly on $\mathbb{R}$, then $\lim\limits_{n\to\infty}\int_{-\infty}^{\infty} f_n(x)\,dx = \int_{-\infty}^{\infty} f(x)\,dx$

(c) If $\{f_n\}$ converges to f uniformly on $\mathbb{R}$, then f is continuous on $\mathbb{R}$.

(d) There exists a sequence of continuous functions $\{f_n\}$ on $\mathbb{R}$, such that $\{f_n\}$ converges to f uniformly on $\mathbb{R}$.

but $\lim\limits_{n\to\infty}\int_{-\infty}^{\infty} f_n(x)\,dx \neq \int_{-\infty}^{\infty} f(x)\,dx$

Ans. **(c,d)**

70. Let V be the be of polynomials over $\mathbb{R}$ of degree less than or equal to n.

For $p(x) = a_0 + a_1 x + ... + a_n x^n$ in V, define a linear transformation $T : V \to V$ by

$(Tp)(x) = a_0 - a_1 x + a_2 x^2 ..... + (-1)^n a_n x^n$. Then which of the following are correct ?

(a) T is one-to-one (b) T is onto (c) T is invertible (d) det $T = 0$

Ans. **(a,b,c)**

71. Let $\{a_n\}$, $\{b_n\}$ be given bounded sequences of positive real numbers. Then (Here $a_n \uparrow b$ means a_n, increase to a as n goes to ∞, similarly, $b_n \downarrow$ b means b_n decreases to b as n goes to ∞)

(a) if $a_n \uparrow a$, then $\sup\limits_{n\ge 1}(a_n b_n) = a\left(\sup\limits_{n\ge 1} b_n\right)$

(b) if $a_n \uparrow a$, then $\sup\limits_{n\ge 1}(a_n b_n) < a\left(\sup\limits_{n\ge 1} b_n\right)$

(c) if $b_n \downarrow b$, then $\inf\limits_{n\ge 1}(a_n b_n) = \left(\inf\limits_{n\ge 1} a_n\right)b$

(d) if $b_n \downarrow b$, then $\inf\limits_{n\ge 1}(a_n b_n) > \left(\inf\limits_{n\ge 1} a_n\right)b$

Ans. **(*)**

72. Let $S \subseteq \mathbb{R}^2$ be defined by $S = \left\{\left(m + \frac{1}{2^{|p|}}, n + \frac{1}{2^{|q|}}\right) : m, n, p, q \in \mathbb{Z}\right\}$. Then

(a) S is discrete in $\mathbb{R}^2$

(b) the set of limit points of S is the set $\{(m,n) : m, n \in \mathbb{Z}\}$

(c) $\mathbb{R}^2 \setminus S$ is connected but not path connected.

(d) $\mathbb{R}^2 \setminus S$ is path connected.

Ans. **(b,d)**

73. Consider a homogeneous system of linear equation $Ax = 0$, where A is an $m \times n$ real matrix and $n > m$. Then which of the following statements are always true ?

(a) $Ax = 0$ has a solution

(b) $Ax = 0$ has no non-zero solution

(c) $Ax = 0$ has a non-zero solution

(d) Dimension of the space of all solutions is at least $n - m$

Ans. **(a,c,d)**

74. Let a,b,c be positive real numbers,

$$D=\left\{(x_1,x_2,x_3)\in\mathbb{R}^3 : x_1^2+x_2^2+x_3^2\le 1\right\},$$

$$E=\left\{(x_1,x_2,x_3)\in\mathbb{R}^3 : \frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}+\frac{x_3^2}{c^2}\le 1\right\}$$

and $A=\begin{bmatrix} a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$, $\det A>1$. Then, for a compactly supported continuous function f on $\mathbb{R}^3$,

Which of the following are correct ?

(a) $\int_D f(Ax)\,dx=\int_E f(x)\,dx$

(b) $\int_D f(Ax)\,dx=\frac{1}{abc}\int_D f(x)\,dx$

(c) $\int_D f(Ax)\,dx=\frac{1}{abc}\int_E f(x)\,dx$

(d) $\int_{\mathbb{R}^3} f(Ax)\,dx=\frac{1}{abc}\int_{\mathbb{R}^3} f(x)\,dx$

Ans. (c,d)

75. Let f: $(0,1)\to\mathbb{R}$ be continuous. Suppose that $|f(x)-f(y)|\le|\sin x - \sin y|$ for all $x, y\in(0, 1)$. Then

(a) f is discontinuous at least at one paint in (0, 1).

(b) f is continuous everywhere on (0, 1), but not uniformly continuous on (0,1).

(c) f is uniformly continuous on (0, 1)

(d) $\lim_{x\to 0^+} f(x)$ exists.

Ans. (c,d)

76. Let $p_n(x)=a_nx^2+b_nx+c_n$ be a sequence of quadratic polynomial where, $a_n,b_n,c_n\in\mathbb{R}$ for all $n\ge 1$ Let $\lambda_0,\lambda_1,\lambda_2$ be distinct real number such that $\lim_{n\to\infty}p_n(\lambda_0)=A_0, \lim_{n\to\infty}p_n(\lambda_1)=A_1$ and $\lim_{n\to\infty}p_n(\lambda_2)=A_2$. Then

(a) $\lim_{n\to\infty}p_n(x)$ exists for all $x\in\mathbb{R}$

(b) $\lim_{n\to\infty}p_n'(x)$ exists for all $x\in\mathbb{R}$

(c) $\lim_{n\to\infty}p_n\left(\frac{\lambda_0+\lambda_1+\lambda_2}{3}\right)$ does not exists

(d) $\lim_{n\to\infty}p_n'\left(\frac{\lambda_0+\lambda_1+\lambda_2}{3}\right)$ does not exists

Ans. (a,b)

77. Define $f:\mathbb{R}^2\to\mathbb{R}^2$ by $f(x,y)=\left(x+2y+y^2+|xy|, 2x+y+x^2+|xy|\right)$ for $(x,y)\in\mathbb{R}^2$. Then

(a) f is discontinuous at (0, 0)

(b) f is continuous at (0, 0) but not differentiable at (0, 0)

(c) f is differentiable at (0, 0)

(d) f is differentiable at (0, 0) and the derivative $Df(0, 0)$ is invertiable

Ans. (c,d)

78. Let $=\left\{(x,y)\in\mathbb{R}^2 : x+y\ne -1\right\}$. Define $f:A\to\mathbb{R}^2$ by $f(x,y)=\left(\frac{x}{1+x+y},\frac{t}{1+x+y}\right)$. Then

(a) the Jacobian matrix of f does not vanish on A

(b) f is infinitely differentiable on A.

(c) f is injective on A

(d) $f(A)=\mathbb{R}^2$

Ans. **(a,b,c)**

79. Which of the following are compact ?

(a) $\{(x,y)\in\mathbb{R}^2:(x-1)^2+(y-2)^2=9\}\cup\{(x,y)\in\mathbb{R}^2:y=3\}$

(b) $\left\{\left(\frac{1}{m},\frac{1}{n}\in\mathbb{R}^2:m,n,\in\mathbb{Z}\setminus\{0\}\right)\right\}\cup\{(0,0)\}\cup\left\{\left(\frac{1}{m},0:m\in\mathbb{Z}\setminus\{0\}\right)\right\}\cup\left\{\left(0,\frac{1}{n}:n\in\mathbb{Z}\setminus\{0\}\right)\right\}$

(c) $\{(x,y,z)\in\mathbb{R}^3:x^2+2y^2-3z^2=1\}$

(d) $\{(x,y,z)\in\mathbb{R}^3:|x|+2|y|-3|z|\le 1\}$

Ans. **(b,c)**

80. Let f be an entire function, Suppose for each $a\in\mathbb{R}$, there exists at least one coefficient c_n is $f(z)=\sum_{n=0}^{\infty}c_n(z-a)^n$, which is zero. Then

(a) $f^{(n)}(0)=0$ for infinitely many $n\ge 0$

(b) $f^{(2n)}(0)=0$ for every $n\ge 0$

(c) $f^{(2n+1)}(0)=0$ for every $n\ge 0$

(d) There exists $K\ge 0$ such that $f^{(n)}(0)=0$ for all $n\ge k$

Ans. **(a,d)**

81. Let $K\subseteq\mathbb{C}$ be a bounded set Let $H(\mathbb{C})$ denote the set of all entire functions and let $C(K)$ denote the set of all continuous functions on K. Consider the restriction map $r:H(\mathbb{C})\to C(K)$ given by $r(f)=f_k$ then r is injective if

(a) K is compact (b) K is connected (c) K is uncountable (d) K is finite

Ans. **(c)**

82. For $z\in\mathbb{C}$, define $f(Z)=\dfrac{e^z}{e^z-1}$. Then

(a) f is entire

(b) the only singularities of f are poles

(c) f has infinite many poles on the imaginary axis

(d) each pole of f is simple

Ans. **(b,c,d)**

83. Let $D=\{z\in\mathbb{C}:|z|<1\}$. Then there exists a holomorphic function $f:D\to\bar{D}$ with $f(0)=0$ with the property

(a) $f'(0)=1/2$ (b) $|f(1/3)|=1/4$

(c) $f(1/3)=1/2$ (d) $|f'(0)|=\sec\left(\frac{\pi}{6}\right)$

Ans. **(a,b)**

84. Let $f(x)=x^4+3x^3-9x^2+7x+27$ and let p be a prime. Let $f_p(x)$ denote the corresponding polynomial with coefficients in $\mathbb{Z}/p\mathbb{Z}$. Then

(a) $f_2(x)$ is irreducible over $\mathbb{Z}/2\mathbb{Z}$ (b) $f(x)$ is irreducible over $\mathbb{Q}$

(c) $f_3(x)$ is irreducible over $\mathbb{Z}/3\mathbb{Z}$ (d) $f(x)$ is irreducible over $\mathbb{Z}$

Ans. **(a,b,d)**

85. Suppose $(F,+,\cdot)$ is finite field with 9 elements. Let $G=(F,+)$ and $H=(F\setminus\{0\},\cdot)$ denote the underlying additive and multiplicative groups respectively. Then

(a) $G\cong(\mathbb{Z}/3\mathbb{Z})\times(\mathbb{Z}/3\mathbb{Z})$

(b) $G\cong(\mathbb{Z}/9\mathbb{Z})$

(c) $H\cong(\mathbb{Z}/2\mathbb{Z})\times(\mathbb{Z}/2\mathbb{Z})\times(\mathbb{Z}/2\mathbb{Z})$

(d) $G\cong(\mathbb{Z}/3\mathbb{Z})\times(\mathbb{Z}/3\mathbb{Z})$ and $H\cong(\mathbb{Z}/8\mathbb{Z})$

Ans. **(a,d)**

86. Consider the multiplicative group G of all the (complex) 2^n-th roots of unity where $n = 0, 1, 2$......
Then

(a) every proper subgroup of G is finite

(b) G has a finite set of generators

(c) G is cyclic

(d) Every finite subgroup of G is cyclic

Ans. **(a,d)**

87. Let R be the ring of all entries function i,e. R is the ring of functions $f:\mathbb{C}\to\mathbb{C}$ that are anlytic at every point of $\mathbb{C}$, with respect to pointwise addition and multiplication. Then

(a) The units in R are precisely the nowhere vanishing entire functions, i.e., $f:\mathbb{C}\to\mathbb{C}$ such that f is entire and $f(\alpha)\neq 0$ for all $\alpha\in\mathbb{C}$

(b) The irreducible elements of R are, up to multiplication by a unit, linear polynomials of the form $z-\alpha$, where $\alpha\in\mathbb{C}$, i,e, if $f\in\mathbb{R}$ is irreducible, then $f(z)=(z-\alpha)g(z)$ for all $z\in\mathbb{C}$ where g is unit in R and $\alpha\in\mathbb{C}$

(c) R is an integral domain

(d) R is unique factorization domain

Ans. **(a,b,c)**

88. We are given a class consisting of 4 boys and 4 girls. A committee that consists of *a* President, *a* Vice-President and *a* Secretary is to be chosen among the 8 students of the class. Let *a* denote the number of ways of choosing the committee in such a way that the committee has at least one boy and at least one girl. Let b denote the number of ways of choosing the committee in such a way that the number of girl is greater than or equal to that of the boys. Then

(a) $a=288$ (b) $b=168$ (c) $a=144$ (d) $b=192$

Ans. **(a,b)**

89. Pick the correct statements

(a) $\mathbb{Q}\left(\sqrt{2}\right)$ and $\mathbb{Q}(i)$ are isomorphic as $\mathbb{Q}$-vector spaces

(b) $\mathbb{Q}\left(\sqrt{2}\right)$ and $\mathbb{Q}(i)$ are isomorphic as fields

(c) $\text{Gal}_{\mathbb{Q}}\left(\mathbb{Q}\left(\sqrt{2}\right)/\mathbb{Q}\right) \cong \text{Gal}_{\mathbb{Q}}\left(\mathbb{Q}(i)/\mathbb{Q}\right)$

(d) $\mathbb{Q}\left(\sqrt{2}\right)$ and $\mathbb{Q}(i)$ are both Galois extensions of $\mathbb{Q}$

Ans. **(a,c,d)**

90. For positive integers m and n, let $F_n = 2^{2^n} + 1$ and $G_m = 2^m - 1$, Which of the following statements are true?

(a) F_n divides G_m whenever $m > n$

(b) GCD (F_n G_m) = 1 whenever $m \neq n$

(c) GCD (F_n , G_m) = 1 whenever $m \neq n$

(d) G_m divides F_n whenever $m < n$

Ans. **(a,c)**

91. Consider a particle of mass m is simple harmonic oscillation about the origin with spring constant k; then for the Lagrangian L and the Hamiltonian H of the system

(a) $L(x,\dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, $H(x,\dot{x}) = \frac{p^2}{2m} + \frac{1}{2}kx^2$; p is generalized momentum

(b) $L(x,\dot{x}) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$ the generalized momentum is $p = m\dot{x}$

(c) $L(x,\dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ and the generalized momentum is $p = m\dot{x}$

(d) $L(x,\dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, $H(x,\dot{x}) = \frac{m\dot{x}^2}{2} + \frac{1}{2}kx^2$

Ans. **(a,c,d)**

92. Consider the boundary value problem $-u''(x) = \pi^2 u(x)$, $x \in (0,1)$, $u(0) = u(1) = 0$. If u and u' are continuous on [0, 1], then

(a) $\int_0^1 u^3(x)\,dx = 0$

(b) $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$

(c) $u'^2(x) + \pi^2 u^2(x) = u'^2(1)$

(d) $\int_0^1 u^2(x)\,dx = \frac{1}{\pi^2}\int_0^1 u'^2(x)\,dx$

Ans. **(b,c,d)**

93. Let $y_1(x)$ and $y_2(x)$ form a complete set of solution to the differential equation

$y'' - 2xy' + \sin\left(e^{2x^2}\right)y = 0, x \in [0,1]$ with

$y_1(0) = 0, y_1'(0) = 1, y_2(0) = 1, y_2'(0) = 1$

Then the Wronskin $W(x)$ of $y_1(x)$ and $y_2(x)$ at $x = 1$ is

(a) e^2 (b) $-e$ (c) $-e^2$ (d) e

Ans. **(b)**

94. Let λ_1, λ_2 be the characteristic numbers and f_1, f_2 the corresponding functions for the homogeneous integral equation $\varphi(x) - \lambda\int_0^1 (xt + 2x^2)\varphi(t)\,dt = 0$. Then

(a) $\lambda_1 = -18 - 6\sqrt{10}, \lambda_2 = -18 + 6\sqrt{10}$

(b) $\lambda_1 = -36 - 12\sqrt{10}, \lambda_2 = -36 + 12\sqrt{10}$

(c) $\int_0^1 f_1(x) f_2(x)\,dx = 1$

(d) $\int_0^1 f_1(x) f_2(x)\,dx = 0$

Ans. (a,d)

95. Consider the function $f(x) = \sqrt{2+x}$ for $x \geq -2$ and the iteration $x_{n+1} = f(x_n); n \geq 0$ for $x_0 = 1$

What are the possible limits of the iteration ?

(a) $\sqrt{2+\sqrt{2+\sqrt{2+.......}}}$

(b) -1

(c) 2

(d) 1

Ans. (a,c)

96. The PDE $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = 0$ is

(a) parabolic and has characteristics $\xi(x, y) = x + 2y, \eta(x, y) = x - 2y$

(b) reducible to the canonical form, $\dfrac{\partial^2 u}{\partial \xi^2} = 0,$ where $\xi(x, y) = x + 2y$

(c) reducible to the canonical form, $\dfrac{\partial^2 u}{\partial \eta^2} = 0,$ where $\eta(x, y) = x + y$

(d) parabolic and has the general solution $u = (x - y) f_1(x + y) + f_2(x - y)$ where f_1, f_2 are arbitrary functions

Ans. (c)

97. Let $u(x, y)$ be an extremal of the function $J(u) = \iint_D \left[\frac{1}{2}u_x^2 + \frac{1}{2}u_y^2 + e^{xy}u\right] dx\,dy$, where D is the open unit disk in $\mathbb{R}^2$. Then u satisfies

(a) $u_{xx} + u_{yy} - e^{x+y} = 0$

(b) $u_{xx} + u_{yy} = e^{xy}$

(c) $u_{xx} + u_{yy} = -e^{xy}$

(d) $\iint_D \left[u_{xx} + y_{yy} - e^{xy}\right] h(x, y)\,dxdy = 0$ for every smooth, h vanishing on the boundary of D.

Ans. (b,d)

98. Consider the iteration

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right), n \geq 0$$

for given $x_0 \neq 0$. Then

(a) x_n converges to $\sqrt{2}$ with rate of convergence 1

(b) x_n converges to $\sqrt{2}$ with rate of convergence 2.

(c) The given iteration is the fixed point iteration for $f(x) = x^2 - 2$

(d) The given iteration is the Newton's method for $f(x) = x^2 - 2$

Ans. (b,d)

99. If $y:[0,\infty) \to (0,\infty]$ is a continuously differentiable function satisfying

$y(t) = y_0 - \int_0^1 y(d)\,ds$ for $t \geq 0$, then

(a) $y^2(t) = y^2(0) + \left(\int_0^t y(s)\,ds\right)^2 - 2y(0)\int_0^t y(s)\,ds$

(b) $y^2(t) = y^2(0) + 2y\int_0^t y^2(s)\,ds$

(c) $y^2(t) = y^2(0) - \int_0^t y(s)\,ds$

(d) $y^2(t) = y^2(0) - 2\int_0^t y^2(s)\,ds$

Ans. (a,d)

100. Let $u(t)$ be a continuously differentiable function taking non-negative values for $t > 0$ satisfying $u'(t) = 3u(t)^{2/3}$ and $u(0) = 0$. Which of the following are possible solutions of the above equation ?

(a) $u(t) = 0$

(b) $u(t) = t^3$

(c) $u(t) = \begin{cases} 0 \text{for } 0 < t < 1 \\ (t-1) \text{ for } t \geq 1 \end{cases}$

(d) $u(t) = \begin{cases} 0 \text{for } 0 < t < 3 \\ (t-3)^3 \text{ for } t \geq 3 \end{cases}$

Ans. (a,b,c,d)

101. Let $u(x, t)$ be the solution of the equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},$$

which tends to zero as $t \to \infty$ and has the value $\cos(x)$ when $t = 0$

Then

(a) $u = \Sigma_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-nt}$ where a_n, b_n are arbitrary constants

(b) $u = \Sigma_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 t}$, where a_n, b_n are non-zero constants

(c) $u = \Sigma_{n=1}^{\infty} a_n \cos(nx + b_n)e^{-nt}$, where a_n, are not all zero and $b_n = 0$ for $n \geq 1$

(d) $u = \Sigma_{n=1}^{\infty} a_n \cos(nx + b_n)e^{-n^2 t}$, where $a_1 \neq 0$, $a_n = 0$ for $n > 1$, and $b_n = 0$ for $n \geq 1$

Ans. (d)

102. Let $xyu = c_1$ and $x^2 + y^2 - 2u = c_2$, where c_1 and c_2 are arbitrary constants, be the first integrals of the PDE $x\left(u+y^2\right)\frac{\partial u}{\partial x} - y\left(u+x^2\right)\frac{\partial u}{\partial y} = \left(x^2 - y^2\right)u$. Then the solution of the PDE $x + y = 0, u = 1$ is given by

(a) $x^3 + y^3 + 2xyu^2 + 2x^2u = 0$

(b) $x^3 + yx^2 + \left(x^2 + xy\right)u = 0$

(c) $x^2 + y^2 + 2\left(xy - 1\right)u + 2 = 0$

(d) $x^2 - y^2 - u\left(x + y - 2\right) - 2 = 0$

Ans. **(c)**

103. Consider the following primal Linear Programming Problem.

max $z = -3x_1 + 2x_2$

Subject to $x_1 \le 3,$

$x_1 - x_2 \le 0,$

$x_1, x_2 \ge 0.$

Which of the following statement are True ?

(a) The primal problem has an optimal solution

(b) The primal problem has an unbiased solution

(c) The dual problem has an unbounded solution

(d) The dual problem has no feasible solution

Ans. **(b,d)**