CSIR-NET-(P YQ) MATHEMATICAL SCIENCE
JUNE-2016-I

PART-B

1. $\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^n$ equal

(a) 1 (b) $e^{-1/2}$ (c) e^{-2} (d) e^{-1}

Ans. (a)

2. Consider the interval (–1, 1) and a sequence $\{\alpha_n\}_{n=1}^{\infty}$ of elements in it. Then,

(a) Every limit point of $\{\alpha_n\}$ is in (–1, 1)

(b) Every limit point of $\{\alpha_n\}$ is in [–1, 1]

(c) The limit points of $\{\alpha_n\}$ can only be in {–1, 0C, 1}

(d) The limit points of $\{\alpha_n\}$ cannot be in {–1, 0, 1}

Ans. (b)

3. Let $F:\mathbb{R}\to\mathbb{R}$ be a monotone function. Then

(a) F has no discontinuities.

(b) F has only finitely many discontinuities.

(c) F can have at most countably many discontinuities.

(d) F can have uncountably many discontinuities.

Ans. (c)

4. Consider the function

$$f(x,y)=\frac{x^2}{y^2},(x,y)\in[1/2,3/2]\times[1/2,3/2]$$

The derivative of the function at (1, 1) along the direction (1, 1) is:

(a) 0 (b) 1 (c) 2 (d) –2

Ans. (a)

5. Consider the improper Riemann integral $\int_0^x y^{-1/2}dy$. This integral is:

(a) continuous in $[0,\infty)$ (b) continuous only in $(0,\infty)$

(c) discontinuous in $(0,\infty)$ (d) discontinuous only in $\left(\frac{1}{2},\infty\right)$

Ans. (a)

6. Which one of the following statements is true for the sequence of functions

$$f_n(x) = \frac{1}{n^2 + x^2}, n = 1, 2, ..., x \in [1/2, 1]?$$

(a) The sequence is monotonic and has 0 as the limit for all $x \in [1/2, 1]$ as $n \to \infty$.

(b) The sequence is not monotonic but has $f(x) = \frac{1}{x^2}$ as the limit as $n \to \infty$.

(c) The sequence is monotonic and has $f(x) = \frac{1}{x^2}$ as the limit as $n \to \infty$.

(d) The sequence is not monotonic but has 0 as the limit

Ans. **(a)**

7. Given a $n \times n$ matrix B define e^B by

$$e^B = \sum_{j=0}^{\infty} \frac{B^j}{j!}$$

Let p be the characteristic polynomial of B. Then the matrix $e^{p(B)}$ is

(a) $I_{n\times n}$ (b) $0_{n\times n}$ (c) $eI_{n\times n}$ (d) $\pi I_{n\times n}$

Ans. **(a)**

8. Let A be a $n\times n$ real symmetric non-singular matrix. Suppose there exists $x \in \mathbb{R}^n$ such that $x'Ax < 0$ Then we can conclude that

(a) $\det(A) < 0$ (b) $B = -A$ is positive definite

(c) $\exists y \in \mathbb{R}^n : y'A^{-1}y < 0$ (d) $\forall y \in \mathbb{R}^n : y'A^{-1}y < 0$

Ans. **(c)**

9. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Let $f : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(v, w) = w^T Av$.

Pick the correct statement from below:

(a) There exists an eigenvector v of A such that Av is perpendicular to v

(b) The set $\{v \in \mathbb{R}^2 \mid f(v, v) = 0\}$ is a nonzero subspace of $\mathbb{R}^2$

(c) If $v, w \in \mathbb{R}^2$ are nonzero vectors such that $f(v, v) = 0 = f(w, w)$, then v is a scalar multiple of w.

(d) For every $v \in \mathbb{R}^2$, there exists a nonzero $w \in \mathbb{R}^2$ such that $f(v, w) = 0$

Ans. **(d)**

10. Let A be a $n \times m$ matrix and b be a $n\times 1$ vector (with real entries). Suppose the equation $Ax = b, x \in \mathbb{R}^m$ admits a unique solution. Then we can conclude that

(a) $m \geq n$ (b) $n \geq m$ (c) $n = m$ (d) $n > m$

Ans. **(b)**

11. Let V be the vector space of all real polynomials of degree ≤ 10. Let $Tp(x) = p'(x)$ for $p \in V$ be a linear transformation from V to V. Consider the basis $\{1, x, x^2, ...x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then

(a) Trace A = I (b) det A = 0

(c) there is no $m \in \mathbb{N}$ such that $A^m = 0$ (d) A has a nonzero eigenvalue

Ans. **(b)**

12. Let $x=(x_1,x_2,x_3), y=(y_1,y_2,y_3)\in\mathbb{R}^3$ be linearly independent.

Let $\delta_1=x_2y_3-y_2x_3, \delta_2=x_1y_3-y_1x_3, \delta_3=x_1y_2-y_1x_2$. If V is the span of x, y, then

(a) $V=\{(u,v,w):\delta_1u-\delta_2v+\delta_3w=0\}$ (b) $V=\{(u,v,w):-\delta_1u+\delta_2v+\delta_3w=0\}$

(c) $V=\{(u,v,w):\delta_1u+\delta_2v-\delta_3w=0\}$ (d) $V=\{(u,v,w):\delta_1u+\delta_2v+\delta_3w=0\}$

Ans. (a)

13. Let $p(x)$ be a polynomial of degree $d\ge 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty}P(n)z^n$ is:

(a) 0 (b) 1 (c) ∞ (d) dependent on d

Ans. (b)

14. Let $P(z)$, $Q(z)$ be two complex non-constant polynomials of degree m,n respectively. The number of roots of P(z) = $P(z)\,Q(z)$ counted with multiplicity is equal to:

(a) min {m,n} (b) max $\{m,n\}$ (c) $m+n$ (d) $m-n$

Ans. (c)

15. The residue of the function $f(z)=e^{e^{-1/z}}$ at $z=0$ is

(a) $1+e^{-1}$ (b) e^{-1} (c) $-e^{-1}$ (d) $1-e^{-1}$

Ans. (c)

16. Let D be the open unit disc in $\mathbb{C}$ and $H(D)$ be the collection of all holomorphic functions on it. Let

$$S=\left\{f\in H(D): f\left(\frac{1}{2}\right)=\frac{1}{2}, f\left(\frac{1}{4}\right)=\frac{1}{4},\ldots f\left(\frac{1}{2n}\right)=\frac{1}{2n},\ldots\right\}$$

and

$$T=\left\{f\in H(D): f\left(\frac{1}{2}\right)=f\left(\frac{1}{3}\right)=\frac{1}{2}, f\left(\frac{1}{4}\right)=f\left(\frac{1}{5}\right)=\frac{1}{4},\ldots, f\left(\frac{1}{2n}\right)=f\left(\frac{1}{2n+1}\right)=\frac{1}{2n},\ldots\right\}$$

Then

(a) Both S, T are singleton sets (b) S is a singleton set but $T=\phi$

(c) T is a singleton set but $S=\phi$ (d) Both S, T are empty

Ans. (b)

17. Which of the following statements is FALSE? There exists an integer such that:

(a) $x\equiv 23 \bmod 1000$ and $x\equiv 45 \bmod 6789$

(b) $x\equiv 23 \bmod 1000$ and $x\equiv 54 \bmod 6789$

(c) $x\equiv 32 \bmod 1000$ and $x\equiv 54 \bmod 9876$

(d) $x\equiv 32 \bmod 1000$ and $x\equiv 44 \bmod 9876$

Ans. (c)

18. Let p be a prime number. How many distinct sub-rings (with unity) of cardinality p does the field F_{p^2} have ?

(a) 0 (b) 1 (c) p (d) p^2

Ans. (b)

19. Let $G = (\mathbb{Z}/25\mathbb{Z})^*$ be the group of units (i.e. the elements that have a multiplicative inverse) in the ring $(\mathbb{Z}/25\mathbb{Z})$. Which of the following is a generator of G?

(a) 3 (b) 4 (c) 5 (d) 6

Ans. **(a)**

20. Let $p \geq 5$ be a prime. Then

(a) $\mathbb{F}_p \times \mathbb{F}_p$ has at least five subgroups of order p

(b) Every subgroup of $\mathbb{F}_p \times \mathbb{F}_p$ is of the form $H_1 \times H_2$ where H_1, H_2 are subgroups of $\mathbb{F}_p$

(c) Every subgroup of $\mathbb{F}_p \times \mathbb{F}_p$ is an ideal of the ring $\mathbb{F}_p \times \mathbb{F}_p$

(d) The ring $\mathbb{F}_p \times \mathbb{F}_p$ is a field

Ans. **(a)**

21. Let y_1 and y_2 be two solutions of the problem

$$\left.\begin{aligned} y''(t) + ay'(t) + by(t) = 0,\ t \in \mathbb{R} \\ y(0) = 0 \end{aligned}\right\}$$

where a and b are real constants. Let w be the Wronskian of y_1 and y_2. Then

(a) $w(t) = 0, \forall t \in \mathbb{R}$

(b) $w(t) = c, \forall t \in \mathbb{R}$ for some positive constant c

(c) w is a nonconstant positive function

(d) There exists $t_1, t_2 \in \mathbb{R}$ such that $w(t_1) < 0 < w(t_2)$

Ans. **(a)**

22. Let $A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix}, x(t) = \begin{vmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{vmatrix}$ and $|x(t)| = \left(x_1^2(t) + x_2^2(t) + x_3^2(t)\right)^{1/2}$. Then any solution of the first order system of the ordinary differential equation

$$\left.\begin{aligned} x'(t) = Ax(t) \\ x(0) = x_0 \end{aligned}\right\} \text{satisfies}$$

(a) $\lim_{t\to\infty} |x(t)| = 0$ (b) $\lim_{t\to\infty} |x(t)| = \infty$ (c) $\lim_{t\to\infty} |x(t)| = 2$ (d) $\lim_{t\to\infty} |x(t)| = 12$

Ans. **(a)**

23. Let a, b, c, d be four differentiable functions defined on $\mathbb{R}^2$. Then the partial differential equation

$$\left(a(x,y)\frac{\partial}{\partial x} + b(x,y)\frac{\partial}{\partial y}\right)\left(c(x,y)\frac{\partial}{\partial x} + d(x,y)\frac{\partial}{\partial y}\right)u = 0 \text{ is}$$

(a) always hyperbolic (b) always parabolic

(c) never parabolic (d) never elliptic

Ans. **(d)**

24. For the Cauchy problem

$$\begin{aligned} u_t - uu_x = 0,\ x \in \mathbb{R}, t > 0 \\ u(x,0) = x,\ x \in \mathbb{R}, \end{aligned}$$

which of the following statements is true ?

(a) The solution exists for all $t > 0$.

(b) The solution u exists for $t < \frac{1}{2}$ and breaks down at $t = \frac{1}{2}$

(c) The solution u exists for $t < 1$ and breaks down at $t = 1$.

(d) The solution u exists for $t < 2$ and breaks down at $t = 2$.

Ans. (c)

25. Let $f(x) = x^2 + 2x + 1$ and the derivative of f at $x = 1$ is approximated by using the central-difference formula

$$f'(1) \approx \frac{f(1+h) - f(1-h)}{2h} \text{ with } h = \frac{1}{2}.$$

Then the absolute value of the error in the approximation of f '(1) is equal to

(a) 1 (b) 1/2 (c) 0 (d) 1/12

Ans. (c)

26. The curve of fixed length l, that joins the points (0, 0) and (1, 0), lies above the x-axis, and encloses the maximum area between itself and the x-axis, is a segment of

(a) a straight line (b) a parabola (c) an ellipse (d) a circle.

Ans. (d)

27. Consider the integral equation

$$y(x) = x^3 + \int_0^x \sin(x-t)\, y(t)\, dt, x \in [0, \pi]$$

Then the value of $y(1)$ is

(a) 19/20 (b) 1 (c) 17/20 (d) 21/20

Ans. (d)

28. Consider the equations of motion for a system

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial \dot{q}_i} = 0, i = 1, 2, 3, ..., n$$

where

$$L = T - V \begin{bmatrix} \text{with } T(t, \dot{q}_i, \dot{q}_i) \text{ as kinetic energy} \\ \text{and } V(t, q_i) \text{ as potential energy} \end{bmatrix}, q_i$$

the generalized coordinates, and q_i the generalized velocities. Then the equations of motion in the form as above are

(a) necessarily restricted to a conservative system but there is no unique choice of L

(b) not necessarily restricted to a conservative system and there is a unique choice of L

(c) necessarily restricted to a conservative system and there is a unique choice of L

(d) not necessarily restricted to a conservative system and there is no unique choice of L

Ans. (*)

29. Hundred (100) tickets are marked 1, 2,...., 100 and are arranged at random. Four tickets are picked from these tickets and are given to four persons A, B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with the smallest value (among A, B, C, D)?

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{12}$

Ans. (d)

30. Let X and Y be independent and identically distributed random variables such that $P(X=0)=P(X=1)=\frac{1}{2}$.

Let $Z=X+Y$ and $W=|X-Y|$. Then which statement is not correct ?

(A) X and W are independent (b) Y and W are independent

(c) Z and W are uncorrelated (d) Z and W are independent.

Ans. (d)

31. Let $\{X_t\}$ and $\{Y_t\}$ be two independent pure birth processes with birth rates λ_1 and λ_2 respectively.

Let $Z_t = X_t + Y_t$. Then

(a) $\{Z_t\}$ is not a pure birth process.

(b) $\{Z_t\}$ is a pure birth process with birth rate $\lambda_1+\lambda_2$.

(c) $\{Z_t\}$ is a pure birth process with birth rate $\min(\lambda_1, \lambda_2)$.

(d) $\{Z_t\}$ is a pure birth process with birth rate $\lambda_1\lambda_2$.

Ans. (b)

32. Let $X_1 \sim N(0,1)$ and let

$$X_2 = \begin{cases} -X_1, & -2 \le X_1 \le 2 \\ X_1, & \text{otherwise} \end{cases}$$

Then identify the correct statement.

(a) Corr $(X_1, X_2) = 1$

(b) X_2 does not have N(0, 1) distribution.

(c) (X_1, X_2) has a bivariate normal distribution.

(d) (X_1, X_2) does not have a bivariate normal distribution.

Ans. (d)

33. Let $X_1,....,X_n$ be a random sample from N $(\theta,1)$, where $\theta\in\{1,2\}$. Then which of the following statements about the maximum likelihood estimator (MLE) of θ is correct?

(a) MLE of θ does not exist.

(b) MLE of θ is $\overline{X}$.

(c) MLE of θ exists but it is not $\overline{X}$.

(d) MLE of θ is an unbiased estimator of θ.

Ans. (c)

34. Let $X_1,....,X_n$ denote a random sample from a $N(\mu,\sigma^2)$ distribution. Let $\mu\in\mathbb{R}$ be known and $\sigma^2\,(>0)$ be unknown. Let $\chi^2_{n,\alpha/2}$ be an upper$(\alpha/2)^{th}$ percentile point of a χ^2_n distribution. Then a 100(1–α)% confidence interval σ^2 for is given by

(a) $\left(\dfrac{\left(\Sigma_1^n X_i^2-\mu^2\right)}{n\chi^2_{n,\alpha/2}},\dfrac{\left(\Sigma_1^n X_i^2-\mu^2\right)}{n\chi^2_{n,1-\alpha/2}}\right)$

(b) $\left(\dfrac{\Sigma_1^n\left(X_i-\mu\right)^2}{(n-1)\chi^2_{(n-1),\alpha/2}},\dfrac{\left(\Sigma_1^n X_i-\mu\right)^2}{(n-1)\chi^2_{(n-1),1-\alpha/2}}\right)$

(c) $\left(\dfrac{\Sigma_1^n\left(X_i-\bar{X}\right)^2}{n\chi^2_{n,\alpha/2}},\dfrac{\Sigma_1^n\left(X_i-\bar{X}\right)^2}{n\chi^2_{n,1-\alpha/2}}\right)$

(d) $\left(\dfrac{\Sigma_1^n\left(X_i-\mu\right)^2}{n\chi^2_{n,\alpha/2}},\dfrac{\Sigma_1^n\left(X_i-\mu\right)^2}{n\chi^2_{n,1-\alpha/2}}\right)$

Ans. **(d)**

35. In the context of testing of statistical hypotheses, which one of the following statements is true?

(a) When testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, the likelihood ratio principle leads to the most powerful test.

(b) When testing a simple hypothesis against an alternative simple hypothesis H_0.
H_1, P [rejecting H_0| H_0 is true] + P [accepting H_0| H_1 is true] = 1.

(c) For testing a simple hypothesis H_0 against an alternative simple hypothesis H_1, randomized test is used to achieve the desired level of the power of the test.

(d) UMP tests for testing a simple hypothesis H_0 against an alternative composite H_1, always exist.

Ans. **(a)**

36. Let be Y_1, Y_2, Y_3 uncorrelated observations with common variance σ^2 and expectations given by

$\mathbb{E}(Y_1)=\beta_1, \mathbb{E}(Y_2)=\beta_2$ and $\mathbb{E}(Y_3)=\beta_1+\beta_2$, where β_1, β_2 are unknown parameters. The best linear unbiased estimator of $\beta_1+\beta_2$

(a) Y_3

(b) Y_1+Y_2

(c) $\frac{1}{2}(Y_1+Y_2+2Y_3)$

(d) $\frac{1}{2}(Y_1+Y_2+Y_3)$

Ans. **(c)**

37. Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ where $\underline{\mu}=(1,1,1)$ and $\sum=\begin{pmatrix}1 & 1 & 1\\ 1 & 3 & c\\ 1 & c & 2\end{pmatrix}$. The value of c such that X_2 and

$-X_1+X_2-X_3$ are independent is

(a) –2 (b) 0 (c) 2 (d) 1

Ans. **(c)**

38. A sample of size $n(\geq 2)$ is drawn without replacement from a finite population of size N, using an arbitrary sampling scheme. Let π_i denote the inclusion probability of the i-th unit and π_{ij}, the joint inclusion probability of units i and $j, 1\leq i<j\leq N$. Which of the following statements is always true?

(a) $\sum_{i=1}^{N}\pi_i = n$

(b) $\sum_{j=1}^{N}\pi_{ij} = n\pi_i, 1\leq i\leq N$

(c) $\pi_{ij}>0$ for all $i, j, 1\leq i<j\leq N$

(d) $\pi_i\pi_j-\pi_{ij}>0$ for all $i, j, 1\leq i<j\leq N$

Ans. **(a)**

39. Consider a series system with two independent components. Let the component lifespan have exponential distribution with density

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & ,\lambda>0, x>0\\ 0, & \text{otherwise}\end{cases}$$

If n observations $X_1, X_2, ..., X_n$ on lifespan of this component are available and

$$\bar{X}=\frac{1}{n}\sum_{1}^{n} X_i,$$

then the maximum likelihood estimator of the reliability of the system is given by

(a) $\left(1-e^{-t/\bar{X}}\right)^2$ (b) $1-\left(1-e^{-t/\bar{X}}\right)^2$ (c) $e^{-2t/\bar{X}}$ (d) $1-e^{-2t/\bar{X}}$

Ans. **(c)**

40. Customers arrive at an ice cream parlour according to a Poisson process with rate 2. Service time distribution has density function

$$f(x)=\begin{cases}3e^{-3x} & ,x>0\\ 0, & x\le 0\end{cases}$$

Upon being served a customer may rejoin the queue with probability 0.4, independently of new arrivals; also a returning customer's service time is the same as that of a new arriving customer. Customers behave independently of each other. Let X(t) = number of customers in the queue at time . Which among the following is correct?

(a) {X(t)} grows without bound with probability 1.

(b) {X(t)} has stationary distribution given by $\pi_k=\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^k$, $k=0,1,2,....$

(c) {X(t)} has stationary distribution given by $\pi_k=(0.1)(0.9)^k$, $k=0,1,2,....$

(d) {X(t)} has stationary distribution given by $\pi_k=(0.4)(0.6)^k$, $k=0,1,2,....$

Ans. **(a)**

PART-C

41. Let $x_1=0$, $x_2=1$, and for $n\ge 3$, define $x_n=\frac{x_{n-1}+x_{n-2}}{2}$. Which of the following is/are true?

(a) $\{x_n\}$ is a monotone sequence (b) $\lim_{n\to\infty} x_n=\frac{1}{2}$

(d) $\{x_n\}$ is a Cauchy sequence (d) $\lim_{n\to\infty} x_n=\frac{2}{3}$

Ans. **(c,d)**

42. Let $\{x_n\}$ be an arbitrary sequence of real numbers. Then

(a) $\Sigma_{n=1}^{\infty}|x_n|^p<\infty$ for some $1<p<\infty$ implies $\Sigma_{n=1}^{\infty}|x_n|^q<\infty$ for any $q>p$.

(b) $\Sigma_{n=1}^{\infty}|x_n|^p<\infty$ for some $1<p<\infty$ implies $\Sigma_{n=1}^{\infty}|x_n|^q<\infty$ for any $1\le q<p$.

(c) Given any $1<p<q<\infty$, there is a real sequence $\{x_n\}$ such that $\Sigma_{n=1}^{\infty}|x_n|^p<\infty$ but $\Sigma_{n=1}^{\infty}|x_n|^q=\infty$

(d) Given any $1<q<p<\infty$, there is a real sequence $\{x_n\}$ such that $\Sigma_{n=1}^{\infty}|x_n|^p<\infty$ but $\Sigma_{n=1}^{\infty}|x_n|^q=\infty$

Ans. **(a,d)**

43. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function and $f(x+1)=f(x)$ for all $x\in\mathbb{R}$. Then

(a) f is bounded above, but not bounded below

(b) f is bounded above and below, but may not attain its bounds

(c) f is bounded above and below and f attains its bounds

(d) f is uniform ly continuous

Ans. **(c,d)**

44. Take the closed interval [0,1] and open interval(1/3, 2/3) . Let K =[0,1]\ (1/3, 2/3). For $x\in[0,1]$ define $f(x)=d(x,K)$ where $d(x,K)=\inf\{|x-y|\,|\,y\in K\}$. Then

(a) $f:[0,1]\to\mathbb{R}$ is differentiable at all points of (0,1)

(b) $f:[0,1]\to\mathbb{R}$ is not differentiable at 1/3 and 2/3

(c) $f:[0,1]\to\mathbb{R}$ is not differentiable at 1/2

(d) $f:[0,1]\to\mathbb{R}$ is not continuous

Ans. **(b,c)**

45. Which of the following is/are true?

(a) (0, 1) with the usual topology admits a metric which is complete

(b) (0, 1) with the usual topology admits a metric which is not complete

(c) [0, 1] with the usual topology admits a metric which is not complete

(d) [0, 1] with the usual topology admits a metric which is complete

Ans. **(a,b,d)**

46. Let V be the span of (1, 1, 1) and (0, 1, 1)$\in\mathbb{R}^3$. Let $u_1=(0,0,1), u_2=(1,1,0)$ and $u_3=(1,0,1)$.

Which of the following are correct?

(a) $(\mathbb{R}^3\setminus V)\cup\{(0,0,0)\}$ is not connected.

(b) $(\mathbb{R}^3\setminus V)\cup\{tu_1+(1-t)u_3 : 0\le t\le 1\}$ is connected.

(c) $(\mathbb{R}^3\setminus V)\cup\{tu_1+(1-t)u_2 : 0\le t\le 1\}$ is connected.

(d) $(\mathbb{R}^3\setminus V)\cup\{(t,2t,2t) : t\in\mathbb{R}\}$ is connected

Ans. **(c,d)**

47. Let A be any set. Let $\mathbb{P}$(A) be the power set of A, that is, the set of all subsets of A; $\mathbb{P}(A)=\{B : B\subseteq A\}$.

Then which of the following is/are true about the set $\mathbb{P}$(A)?

(a) $\mathbb{P}(A)=\Phi$ for some A

(b) $\mathbb{P}$(A)is a finite set for some A

(c) $\mathbb{P}$(A) is a countable set for some A

(d) $\mathbb{P}$(A) is a uncountable set for some A

Ans. **(b,d) or (b,c,d)**

48. Which of the following functions is/are uniformly continuous on the interval (0,1) ?

(a) $\frac{1}{x}$

(b) $\sin\frac{1}{x}$

(c) $x\sin\frac{1}{x}$

(d) $\frac{\sin x}{x}$

Ans. **(c,d)**

49. Define f on [0, 1] by

$$f(x)=\begin{cases} x^2 & \text{if } x \text{ is rational} \\ x^3 & \text{if } x \text{ is irrational} \end{cases}. \text{ Then}$$

(a) f is not Riemann integrable on [0, 1].

(b) f is Riemann integrable and $\int_0^1 f(x)\,dx=\frac{1}{4}$

(c) f is Riemann integrable and $\int_0^1 f(x)\,dx=\frac{1}{3}$

(d) $\frac{1}{4}=\underline{\int_0^1} f(x)\,dx<\overline{\int_0^1} f(x)\,dx=\frac{1}{3}$, where $\underline{\int_0^1} f(x)\,dx$ and $\overline{\int_0^1} f(x)\,dx$ are the lower and upper Riemann integrals of f.

Ans. **(a,d)**

50. Let V be the vector space of all complex polynomials p with deg $p \le n$. Let T: $V\to V$ be the map $(Tp)(x)=p'(1), x\in\mathbb{C}$. Which of the following are correct?

(a) dim Ker $T = n$

(b) dim range $T = 1$

(c) dim Ker $T = 1$

(d) dim range $T = n+1$

Ans. **(a,b)**

51. Consider the real vector space V of polynomials of degree less than or equal to d. For $p\in V$ define

$$\|p\|_k=\max\left\{|p(0)|,\left|p^{(1)}(0)\right|,\ldots,\left|p^{(k)}(0)\right|\right\}$$

where $p^{(i)}(0)$ is the i^{th} derivative of p evaluated at 0. Then $\|p\|_k$ defines a norm on V if and only if

(a) $k\ge d-1$

(b) $k<d$

(c) $k\ge d$

(d) $k<d-1$

Ans. **(c)**

52. Let A, B be $n\times n$ real matrices such that det $A>0$ and det $B<0$. For $0\le t\le 1$ consider $C(t)=tA+(1-t)B$, Then

(a) C(t) is invertible for each $t\in[0,1]$

(b) There is a $t_0\in(0,1)$ such that $C(t_0)$ is not invertible.

(c) $C(t)$ is not invertible for each $t\in[0,1]$.

(d) C (t) is invertible for only finitely many $t\in[0,1]$.

Ans. **(b)**

53. Let A be an $n\times n$ real matrix. Pick the correct answer(s) from the following

(a) A has at least one real eigenvalue.

(b) For all nonzero vectors $v,w\in\mathbb{R}^n$, $(Aw)^T(Av)>0$

(c) Every eigenvalue of A^TA is a nonnegative real number

(d) $I+A^TA$ is invertible

Ans. **(c,d)**

54. Let $\{a_1,...,a_n\}$ and $\{b_1,...,b_n\}$ be two bases of $\mathbb{R}^n$. Let P be an $n \times n$ matrix with real entries such that $Pa_i = b_i$ $i = 1,2,....,n$. Suppose that every eigenvalue of P is either –1 or 1. Let Q = I +2P. Then which of the following statements are true?

(a) $\{a_i + 2b_i \mid i = 1,2,...,n\}$ is also a basis of V

(b) Q is invertible.

(c) Every eigenvalue of Q is either 3 or –1 .

(d) det Q > 0 if det P > 0.

Ans. (a,b,c,d) or (b,c,d)

55. Let T be a $n \times n$ matrix with the property $T^n = 0$. Which of the following is/are true?

(a) T has n distinct eigenvalues.

(b) T has one eigenvalue of multiplicity n.

(c) 0 is an eigenvalue of T.

(d) T is similar to a diagonal matrix

Ans. (c) or (b,c)

56. Let A be an $n \times n$ matrix with real entries. Define $\langle x, y \rangle_A := \langle Ax, Ay \rangle$, $x, y \in \mathbb{R}^n$. Then $\langle x, y \rangle_A$ defines an inner-product if and only if

(a) Ker $A = 0$

(b) Rank $A = n$

(c) All eigenvalues of A are positive

(d) All eigenvalues of A are non-negative

Ans. (a,b)

57. Suppose $\{v_1,...,v_n\}$ are unit vectors in $\mathbb{R}^n$ such that

$$\|v\|^2 = \sum_{i=1}^{n} \left|\langle v_i, v \rangle\right|^2, \forall v \in \mathbb{R}^n$$

Then decide the correct statements in the following

(a) $v_1,....,v_n$ are mutually orthogonal.

(b) $\{v_1,....,v_n\}$ is a basis for $\mathbb{R}^n$

(c) $v_1,....,v_n$ are not mutually orthogonal.

(d) Atmost n–1of the elements in the set $\{v_1,....,v_n\}$ can be orthogonal

Ans. (a,b)

58. Let V = { $f : [0,1] \to \mathbb{R} \mid f$ is a polynomial of degree less than or equal to n}

Let $f_j(x) = x^j$ for $0 \le j \le n$ and let A be the $(n+1) \times (n+1)$ matrix given by $a_{ij} = \int_0^1 f_i(x) f_j(x) dx$. Then which of the following is/are true ?

(a) dim $V = n$

(b) dim $V > n$

(c) A is nonnegative definite, i.e., for all $v \in \mathbb{R}^n, \langle Av, v \rangle \geq 0$.

(d) det A > 0

Ans. (b,d) or (b,c,d)

59. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function. Suppose that $f = u + iv$ where u,v are the real and imaginary parts of f respectively. Then f is constant if

(a) $\{u(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded.

(b) $\{v(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded.

(c) $\{u(x, y) + v(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded.

(d) $\{u^2(x, y) + v^2(x, y) : z = x + iy \in \mathbb{C}\}$ is bounded.

Ans. (a,b,c,d)

60. Let $A = \{z \in \mathbb{C} \mid |z| > 1\}$, $B = \{z \in \mathbb{C} \mid z \neq 0\}$. Which of the following are true ?

(a) There is a continuous onto function $f : A \to B$

(b) There is a continuous one to one function $f : B \to A$

(c) There is a nonconstant analytic function $f : B \to A$

(d) There is a nonconstant analytic function $f : A \to B$

Ans. (a,b,d)

61 Let $H = \{z = x + iy \in \mathbb{C} : y > 0\}$ be the upper half plane and $D = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc. Suppose that f is a Mobius transformation, which maps H conformally onto D. Suppose that $f(2i) = 0$. Pick each correct statement from below.

(a) f has a simple pole at $z = -2i$

(b) f satisfies $f(i)\overline{f(-i)} = 1$

(c) f has an essential singularity at $z = -2i$

(d) $|f(2 + 2i)| = \frac{1}{\sqrt{5}}$

Ans. (a,b,d)

62. Consider the function

$$F(z) = \int_1^2 \frac{1}{(x - z)^2} dx, \ \mathrm{Im}(z) > 0$$

Then there is a meromorphic function $G(z)$ on $\mathbb{C}$ that agrees with $F(z)$ when $Im(z) > 0$, such that

(a) 1, ∞ are poles of $G(z)$

(b) 0,1, ∞ are poles of $G(z)$

(c) 1, 2 are poles of $G(z)$

(d) 1, 2 are simple poles of $G(z)$

Ans. (c,d)

63. Consider the integral $A=\int_0^1 x^n(1-x)^n\,dx$. Pick each correct statement from below.

(a) A is not a rational number (b) $0<A\le 4^{-n}$

(c) A is a natural number. (d) A^{-1} is a natural number

Ans. (*)

64. Let G be a finite abelian group of order n. Pick each correct statement from below.

(a) If d divides n, there exists a subgroup of G of order d.

(b) If d divides n, there exists an element of order d in G.

(c) If every proper subgroup of G is cyclic, then G is cyclic.

(d) If H is a subgroup of G, there exists a subgroup N of G such that G/N$\cong$H.

Ans. (a,d)

65. Consider the symmetric group S_{20} and its subgroup A_{20} consisting of all even permutations. Let H be a 7-Sylow subgroup of A_{20}. Pick each correct statement from below:

(a) $|H| = 49$

(b) H must be cyclic

(c) H is a normal subgroup of A_{20}

(d) Any 7-Sylow subgroup of S_{20} is a subset of A_{20}

Ans. (a,d)

66. Let p be a prime. Pick each correct statement from below. Up to isomorphism,

(a) there are exactly two abelian groups of order p^2

(b) there are exactly two groups of order p^2

(c) there are exactly two commutative rings of order p^2

(d) there is exactly one integral domain of order p^2

Ans. (a,bd)

67. Let R be a commutative ring with unity, such that $R[X]$ is a UFD. Denote the ideal (X) of $R[X]$ by l. Pick each correct statement from below:

(a) I is prime

(b) If I is maximal, then $R[X]$ is a PID

(c) If $R[X]$ is a Euclidean domain, then I is maximal

(d) If $R[X]$ is a PID, then it is a Euclidean domain

Ans. (a,b,c,d)

68. Let $f(x)\in\mathbb{Z}[x]$ be a polynomial of degree ≥ 2. Pick each correct statement from below:

(a) If $f(x)$ is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{Q}[x]$

(b) If $f(x)$ is irreducible in $\mathbb{Q}[x]$, then it is irreducible in $\mathbb{Z}[x]$

(c) If $f(x)$ is irreducible in $\mathbb{Z}[x]$, then for all primes the reduction $\overline{f(x)}$ of $f(x)$ modulo p is irreducible in $\mathbb{F}_p[x]$.

(d) If $f(x)$ is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{R}[x]$

Ans. (a,b,d)

69. Consider the smallest topology τ on $\mathbb{C}$ in which all the singleton sets are closed. Pick each correct statement from below:

(a) $(\mathbb{C}, \tau)$ is Hausdorff. (b) $(\mathbb{C}, \tau)$ is compact (c) $(\mathbb{C}, \tau)$ is connected. (d) $\mathbb{Z}$ is dense in $(\mathbb{C}, \tau)$

Ans. **(b,c,d)**

70. Let $\{X_\alpha\}_{\alpha\in I}$ be discrete topological spaces and let $X = \prod_{\alpha\in I} X_\alpha$. From the statements given below, pick each statement that implies that the product topology on X equals the discrete topology on X

(a) I is finite.

(b) I is countably infinite and X_α are singletons for all but finitely many α

(c) I is uncountably infinite and X_α are singletons for all but finitely many α

(d) I is infinite and X_α are infinite for all α

Ans. **(a,b,c)**

71. Let $y:\mathbb{R}\to\mathbb{R}$ be a solution of the ordinary differential equation, $2y''+3y'+y=e^{-3x}$, $x\in\mathbb{R}$ satisfying $\lim\limits_{x\to\infty} e^x y(x)=0$. Then

(a) $\lim\limits_{x\to\infty} e^{2x} y(x)=0$ (b) $y(0)=\dfrac{1}{10}$

(c) y is a bounded function on $\mathbb{R}$ (d) $y(1)=0$

Ans. **(a,b)**

72. For $\lambda\in\mathbb{R}$, consider the differential equation

$$y'(x)=\lambda\sin(x+y(x)),\ y(0)=1$$

Then this initial value problem has :

(a) no solution in any neighbourhood of 0.

(b) a solution in $\mathbb{R}$ if $|\lambda|<1$

(c) a solution in a neighbourhood of 0.

(d) a solution in $\mathbb{R}$ only if $|\lambda|>1$

Ans. **(b,c)**

73. The problem

$$\left.\begin{aligned} -y''+(1+x)y=\lambda y, \quad x\in(0,1) \\ y(0)=y(1)=0 \end{aligned}\right\}$$

has a non zero solution

(a) for all $\lambda<0$ (b) for all $\lambda\in[0,1]$

(c) for some $\lambda\in(2,\infty)$ (d) for a countable number λ's

Ans. **(c,d)**

74. Let $u:\mathbb{R}\times[0,\infty)\to\mathbb{R}$ be a solution of the initial value problem

$$\left.\begin{aligned} u_{tt}-u_{xx}&=0, && \text{for } (x,t)\in\mathbb{R}\times(0,\infty) \\ u(x,0)&=f(x), && x\in\mathbb{R} \\ u_t(x,0)&=g(x), && x\in\mathbb{R} \end{aligned}\right\}$$

Suppose $f(x)=g(x)=0$ for $x\notin[0,1]$, then we always have

(a) $u(x,t)=0$ for all $(x,t)\in(-\infty,0)\times(0,\infty)$ (b) $u(x,t)=0$ for all $(x,t)\in(1,\infty)\times(0,\infty)$

(c) $u(x,t)=0$ for all (x,t) satisfying $x+t<0$ (d) $u(x,t)=0$ for all (x,t) satisfying $x-t>1$

Ans. **(c,d)**

75. Consider the Cauchy problem for the Eikonal equation

$$p^2 + q^2 = 1;\ p \equiv \frac{\partial u}{\partial x},\ q \equiv \frac{\partial u}{\partial y}$$

$u(x, y) = 0$ on $x + y = 1$, $(x, y) \in \mathbb{R}^2$ Then

(a) The Charpit's equations for the differential equation are

.
$$\frac{dx}{dt} = 2p;\ \frac{dy}{dt} = 2q;\ \frac{du}{dt} = 2;\ \frac{dp}{dt} = -p;\ \frac{dq}{dt} = -q$$

(b) The Charpit's equations for the differential equation are

$$\frac{dx}{dt} = 2p;\ \frac{dy}{dt} = 2q;\ \frac{du}{dt} = 2;\ \frac{dp}{dt} = 0;\ \frac{dq}{dt} = 0$$

(c) $u\left(1, \sqrt{2}\right) = \sqrt{2}$

(d) $u\left(1, \sqrt{2}\right) = 1$

Ans. **(b,d)**

76. Let $H(x)$ be the cubic Hermite interpolation of $f(x) = x^4 + 1$ on the interval $I = [0,1]$ interpolating at $x = 0$ and $x = 1$. Then

(a) $\max_{x \in I} \left| f(x) - H(x) \right| = \frac{1}{16}$

(b) The maximum of $\left| f(x) - H(x) \right|$ is attained at $x = \frac{1}{2}$

(c) $\max_{x \in I} \left| f(x) - H(x) \right| = \frac{1}{21}$

(d) The maximum of $\left| f(x) - H(x) \right|$ is attained at $x = \frac{1}{4}$

Ans. **(a,b)**

77. Let $f : [0,3] \to \mathbb{R}$ be defined by $f(x) = \left| 1 - |x - 2| \right|$ where $|.|$ denotes the absolute value. Then for the numerical approximation of $\int_0^3 f(x)\,dx$, which of the following statements are true?

(a) The composite trapezoid rule with three equal subintervals is exact

(b) The composite midpoint rule with three equal subintervals is exact

(c) The composite trapezoid rule with four equal subintervals is exact

(d) The composite midpoint rule with four equal subintervals is exact

Ans. **(a,b)**

78. Let u be the solution of the boundary value problem

$u_{xx} + u_{yy} = 0$ for $0 < x, y < \pi$

$u(x, 0) = 0 = u(x, \pi)$ for $0 \le x \le \pi$

$u(0, y) = 0,\ u(\pi, y) = \sin y + \sin 2y$ for $0 \le y \le \pi$

then

(a) $u\left(1,\frac{\pi}{2}\right)=\left(\sinh(\pi)\right)^{-1}\sinh(1)$

(b) $u\left(1,\frac{\pi}{2}\right)=\left(\sinh(1)\right)^{-1}\sinh(\pi)$

(c) $u\left(1,\frac{\pi}{4}\right)=\left(\sinh(\pi)\right)^{-1}\left(\sinh(1)\right)\frac{1}{\sqrt{2}}+\left(\sinh(2\pi)\right)^{-1}\sinh(2)$

(d) $u\left(1,\frac{\pi}{4}\right)=\left(\sinh(1)\right)^{-1}\left(\sinh(\pi)\right)\frac{1}{\sqrt{2}}+\left(\sinh(2)\right)^{-1}\sinh(2\pi)$

Ans. (a,c)

79. Consider the Runge-Kutta method of the form

$y_{n+1}=y_n+ak_1+bk_2$

$k_1=hf\left(x_n,y_n\right)$

$k_2=hf\left(x_n+\alpha h,y_n+\beta k_1\right)$

to approximate the solution of the initial value problem

$y'(x)=f\left(x,y(x)\right),y\left(x_0\right)=y_0$

Which of the following choices of a, b, α and β yield a second order method

(a) $a=\frac{1}{2},b=\frac{1}{2},\alpha=1,\beta=1$

(b) $a=1,b=1,\alpha=\frac{1}{2},\beta=\frac{1}{2}$

(c) $a=\frac{1}{4},b=\frac{3}{4},\alpha=\frac{2}{3},\beta=\frac{2}{3}$

(d) $a=\frac{3}{4},b=\frac{1}{4},\alpha=1,\beta=1$

Ans. (a,c)

80. The curve $y=y(x)$ passing through the point $\left(\sqrt{3},1\right)$ and defined by the following property(Voltera integral equation of the first kind) $\int_0^y\frac{f(v)\,dv}{\sqrt{y-v}}=4\sqrt{y}$, where $f(y)=\sqrt{1+\frac{1}{y'^2}}$, is the part of a

(a) straight line (b) circle (c) parabola (d) cycloid

Ans. (a)

81. Let $y=y(x)$ be the extremal of the functional $I\left[y(x)\right]=\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$,

subject to the condition that the left end of the extremal moves along $y=x^2$ while the right end moves along $x-y=5$. Then the

(a) shortest distance between the parabola and the straight line is $\left(\frac{19\sqrt{2}}{8}\right)$

(b) slope of the extremal at(x,y) is $\left(-\frac{3}{2}\right)$

(c) point $\left(\frac{3}{4},0\right)$ lies on the extremal

(d) extremal is orthogonal to the curve $y=\frac{x}{2}$

Ans. (a,c)

82. A particle of unit mass moves in the direction of -axis such that it has the Lagrangian

$$L=\frac{1}{12}\dot{x}^4+\frac{1}{2}x\dot{x}^2-x^2$$

Let $Q=\dot{x}^2\ddot{x}$ represent a force (not arising from a potential) acting on the particle in the x-direction. If $x(0)=1$ and $\dot{x}(0)=1$, then the value of $\dot{x}$ is

(a) some non-zero finite value at $x=0$.

(b) 1 at $x=1$

(c) $\sqrt{5}$ at $x=\frac{1}{2}$

(d) 0 at $x=\sqrt{\frac{3}{2}}$

Ans. (b,c,d)