TIER II: Syllabus-TGT (Mathematics)

REAL NUMBERS

- Review of representation of natural numbers, integers, and rational numbers on the number line. Rational numbers as recurring/ terminating decimals. Operations on real numbers.
- Examples of non-recurring/non-terminating decimals. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}$, $\sqrt{3}$, and their representation on the number line. Explaining that every real number is represented by a unique point on the number line and conversely, viz. every point on the number line represents a unique real number.
 - Definition of nth root of a real number.
 - Rationalization, real numbers of the type $\frac{1}{a+b\sqrt{x}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$ their combinations where x and y are natural number and a and b are integers.
- Laws of exponents with integral powers. Rational exponents with positive real bases Fundamental Theorem of Arithmetic statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$
 - Number theory, squences and patterns, Triangular Number, Hexagonal Number, square Numbers, Cube Numbers, patterns, Shape sequences, super cells, palindromic patterns. Kaprekar Constant .clock and Calender Number, collatz conjecture Bhramagupta's method of Computation.

POLYNOMIALS

- Definition of a polynomial in one variable, with examples and counter examples.
- Coefficients of a polynomial, terms of a polynomial and zero polynomial.
- Degree of a polynomial. Constant, linear, quadratic and cubic polynomials. Monomials, binomials, trinomials. Factors and multiples.
- Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.
- Remainder Theorem with examples, Factor Theorem.
- Factorization of $ax^2 + bx + c$, $a \ne 0$ where a, b and c are real numbers, and of cubic polynomials using the Factor Theorem.
- The algebraic expressions and identities. Verification of identities:

$$(x + y + z)^{2} = x^{2} + y^{2} + z^{2} + 2xy + 2yz + 2zx$$

$$(x \pm y)^{3} = x^{3} \pm y^{3} \pm 3xy (x \pm y)$$

$$x^{3} \pm y^{3} = (x \pm y) (x^{2} \mp xy + y^{2})$$

$$x^{3} + y^{3} + z^{3} - 3xyz = (x + y + z) (x^{2} + y^{2} + z^{2} - xy - yz - zx)$$

And their use in factorization of polynomials.

MATRICES

R,R2,R3 as vector spaces over R and concept of Rn.Standard basis for each of them. Linear Independence and examples of different bases.Subspaces of R2, R3. Translation, Dilation, Rotation, Reflection in a point, line and plane. Matrix form of basic geometric transformations. Interpretation of eigen values and eigen vectors for such transformations and eigen spaces as invariant subspaces. Matrices in diagonal form. Reduction to diagonal form upto matrices of order 3. Computation of matrix inverses using elementary row operations.Rank of matrix, Solutions of a system of linear equations using matrices

LINEAR EQUATIONS IN TWO VARIABLES

Linear equations in one variable. Introduction to the equation in two variables. Focus on linear equations of the type ax + by + c=0. Explain that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers, plotting them and showing that they lie on a line.

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency. Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

QUADRATIC EQUATIONS

Standard form of a quadratic equation ax2 + bx + c = 0, (a $\neq 0$). Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

ARITHMETIC PROGRESSIONS

Arithmetic Progression, nth term and sum of the first n terms of A.P. and their application in solving daily life problems.

COORDINATE GEOMETRY

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations. Graphs of linear equations. Distance formula. Section formula (internal division) Area of Triangle.

INTRODUCTION TO EUCLID'S GEOMETRY

History - Geometry in India and Euclid's geometry. Euclid's method of formalizing observed phenomenon into rigorous Mathematics with definitions, common/obvious notions, axioms/postulates and theorems. The five postulates of Euclid. Showing the relationship between axiom and theorem, for example: (Axiom) 1. Given two distinct points, there exists one and only one line through them. (Theorem) 2. (Prove) Two distinct lines cannot have more than one point in common.

LINES AND ANGLES

- If a ray stands on a line, then the sum of the two adjacent angles so formed is 180 degrees and the converse.
- If two lines intersect, vertically opposite angles are equal.
- Lines which are parallel to a given line are parallel.

TRIANGLES

- Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
- Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
- Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
- Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle. (RHS Congruence)
- The angles opposite to equal sides of a triangle are equal.
- The sides opposite to equal angles of a triangle are equal.

- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
- If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
- If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
- If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
- If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

QUADRILATERALS

- The diagonal divides a parallelogram into two congruent triangles.
- In a parallelogram opposite sides are equal, and conversely.
- In a parallelogram opposite angles are equal, and conversely.
- A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.
- In a parallelogram, the diagonals bisect each other and conversely.
- In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and in half of it and (motivate) its converse.

CIRCLES

- Equal chords of a circle subtend equal angles at the center and (motivate) its converse.
- The perpendicular from the center of a circle to a chord bisects the chord and conversely, the line drawn through the center of a circle to bisect a chord is perpendicular to the chord.
- Equal chords of a circle (or of congruent circles) are equidistant from the center (or their respective centers) and conversely.
- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle.
- The sum of either of the pair of the opposite angles of a cyclic quadrilateral is 180° and its converse.
- Tangent to a circle at, point of contact
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- The lengths of tangents drawn from an external point to a circle are equal.

AREAS

Area of a triangle using Heron's formula, Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120°.

SURFACE AREAS AND VOLUMES

Surface areas and volumes of spheres (including hemispheres) and right circular cones. Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones. Area of a Triangle using Hero's formula and its application in finding the area of a quadrilateral.

STATISTICS

Bar graphs, histograms (with varying base lengths), and frequency polygons. Mean, median and mode of grouped and ungrouped data, infrographics.,presentation of data, tabular form.

PROBABILITY

Classical definition of probability. Simple problems on finding the probability of an event. elementary probability and basic law. Discrete and continuous random variable.

TRIGONOMETRY

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.

HEIGHTS AND DISTANCES:

Angle of elevation, Angle of Depression. Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30° , 45° , and 60°

CALCULUS.

Sets. Functions and their graphs: polynomial, sine, cosine, exponential and logarithmic functions. Step function, Limits and continuity. Differentiation, Methods of differentiation like Chain rule, Product rule and Quotient rule. Second order derivatives of above functions. Integration as reverse process of differentiation. Integrals of the functions introduced above.

INEQUALITIES

Elementary Inequalities, Absolute value, Inequality of means, Cauchy – Schwarz Inequality, Tchebychef's Inequality.